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RESEARCHES ON THE PHYSICAL THEORY OF METEOR PHENOMENA

I.
THEORY OF THE FORMATION OF METEOR CRATERS

II.
THE POSSIBLE CONSEQUENCES OF THE COLLISIONS OF METEORS IN SPACE

BY

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I. Theory of the Formation of Meteor Craters [1]).

Abstract.

The theory of meteor impact, having a direct bearing on the Arizona crater, and the craters of the Moon (at least partly), is also important from the standpoint of meteor cosmogony (see next paper of this Series).

Whereas during the motion of the meteor projectile in the terrestrial atmosphere the pressure seldom exceeds the plastic limit of a solid body, at the impact with cosmic velocity into a solid medium the pressure, as a rule, exceeds the plastic limit. The problem is therefore a case of the impact of a liquid drop into a liquid medium. A mathematical model, supposed to represent well the main qualitative features of the phenomenon, and to yield quantitative estimates to a close order of magnitude has been worked out.

The main feature is the flattening of the projectile during the process of penetration (like a "dum dum" bullet); the shape of the bottom of crater No. 4 at Kaali järv, Estonia, — the only bottom of a meteor crater hitherto explored, — seems to support this feature.

Two different methods for estimating the mass of the projectile, from penetration, and from the mass of crushed and ejected rock, are proposed. In the case of the Arizona crater both methods point accordantly to a mass of about four million tons. This mass, however, could never persist in one piece, even if it originally was one compact body. At the best, the mass broken into pieces of the order of a few metres' diameter each, may be spread at slightly different depths over a projected area of about three to four hundred metres' diameter, placed about one hundred metres excentrically at the true bottom of the crater.

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1) Researches on the Physical Theory of Meteor Phenomena, Series of Papers. Below referred to as paper A of the Series: Ernst Öpik, Atomic Collisions and Radiation of Meteors, Acta et Comm. Univ. Tartuensis **XXVI.2**, 1933; also Harvard Reprint No. **100**.

The problem of meteor impact into solid ground is too complicated for accurate mathematical treatment; also, there always exists the danger of an illegal extension into the unusual conditions of the collision of physical properties, known to be valid in ordinary conditions. Nevertheless, with the purpose of getting reasonable upper and lower estimates of the mass of the projectile, certain model calculations may be made, with due allowance for the fundamental laws of physics.

a. Minimum mass of the projectile.

The mechanical work required to lift up the walls of the crater, throwing out the fragments, and shattering and pulverizing the rocks, must represent a very small fraction of the total energy developed at impact. By setting this amount of work equal to the kinetic energy of the meteor, we get a minimum estimate of its mass. This method seems to have been first used by the writer [1]), and has been applied later independently by different authors (Gifford, and others).

For the Arizona crater the estimate is as follows. Dimensions (Barringer): diameter 1200 metres, height of walls outside 36—49 metres, depth of base of solid rock 320 metres below ground level; probable zenith angle of incidence of projectile 20^0; depth of penetration of projectile 320 sec $20^0 = 340$ metres; mass of material thrown out or shattered etc. (0.36 km^3) 1.0×10^9 tons; this we call later "the mass affected". Judging from the distance at which fragments were found it appears safe to assume the mechanical work equivalent to a lifting of all the mass involved to a height of 1200 metres or 1.2×10^8 erg/gr. The work of shattering may be estimated at about 6×10^6 erg/gr (from compressibility and crushing strength), or a small fraction of the whole. For a velocity of impact $w_0 = 20$ km/sec, the

1) E. Öpik, Remarque sur la théorie météorique des cirques lunaires, Bull. de la Société Russe des Amis de l'Etude de l'Univers, No. 3 (21), pp. 125—134, 1916; in Russian with French abstract.

kinetic energy is 2×10^{12} erg/gr, and the minimum mass of the projectile $\frac{1.2 \times 10^{8}}{2 \times 10^{12}} = 6 \times 10^{-5}$ of the mass affected, or 60 000 tons (equivalent diameter of iron sphere 24 metres or 1/50 of the diameter of the crater). For $w_0 = 60$ km/sec, the minimum mass is 7×10^{-6} of the mass affected or 7000 tons (iron diameter 12 metres).

b. Penetration of the projectile.

The "aerodynamic" pressure at the penetration of a meteor into rock is of the order of 10^{7}—10^{8} atmospheres, or more than 1000 times the plastic limit of steel; no doubt all solid materials under such pressures must behave like liquids; thus the problem of meteor impact is the case of the impact of a liquid drop of given density δ into a liquid medium of density ϱ. The forces of inertia are the only forces of importance except when the velocity is very low. For the impact of iron ($\delta = 7.8$) into stone ($\delta = 2.6$) iron is fluid for velocities exceeding 1.6 km/sec. (aerodynamic pressure = plastic limit).

The effective aerodynamic resistance is only approximately given by $C\varrho w^2$ (per unit of cross section normal to the direction of motion), and depends upon the ratio $\varrho : \delta$ [1]; the pressure is actually a complicated hydrodynamic variable; because of the fluidity of the material there does not exist any definite surface of resistance; the impinging body changes in shape, the main feature of the change being evidently a flattening, or broadening at right angles to the direction of motion, with a possible breaking into smaller pieces [2]. Hydrodynamic stream lines are formed both in the medium, and in the projectile itself. On the other hand, complications which are known at velocities near the velocity of sound do not exist in the present case. At the same time, the propagation of the shock wave, in the medium and inside the projectile, goes on with an initial speed comparable with the velocity of the projectile, much faster than the "ordinary" velocity of sound.

Taking into account, however, that the expression $C\varrho w^2$ for the aerodynamic resistance follows directly from the law of

1) When $\varrho : \delta$ is small, as in the case of iron impact into stone, the formula holds well.

2) Process similar to what happens to a "dum dum" rifle bullet.

conservation of momentum, we feel safe in using it in the following model for calculations within a close order of magnitude, with the understanding that w is a certain effective relative velocity of the projectile, and that the deceleration produced by the imaginary aerodynamic resistance refers to the centre of the mass.

In our rough calculations aiming at the order of magnitude only, any original shape of the projectile if not too "unusual" is as good as another. We assume a cylinder of height equal to diameter $2R_0$, moving axially. Denoting the penetration by x, we assume that the flattening projectile remains a cylinder of radius R, and height H; assuming an incompressible fluid [1]), the constancy of volume gives:

$$R^2 H = 2R_0^3 \quad \ldots \ldots \ldots \quad (1).$$

Let the velocity at the front surface be w', the velocity of the centre of mass be w, and (with a linear gradient of velocity) the velocity at the rear surface be $2w - w'$. For the sake of simplicity, we assume that such a gradient of velocity in the projectile exists from the first moment of contact with the medium. The loss of momentum per unit of time and unit cross section of the projectile we assume equal to

$$p' = \tfrac{1}{2}\varrho w'^2 + k \quad {}^{2)},$$

where k is the crushing strength of the medium which becomes important only when the velocity drops down to about 1 km/sec. Let the velocity of sidewise expansion be

$$w'' = \frac{dR}{dt} \quad \ldots \ldots \ldots \quad (2); \text{ also:}$$

$$-2(w - w') = \frac{dH}{dt} \quad \ldots \ldots \quad (3).$$

From (1), (2), and (3) we get:

$$w'' = \frac{R}{H}(w - w') \quad \ldots \ldots \quad (4).$$

The pressure on the lateral surface of our cylinder we assume equal to

$$p'' = \tfrac{1}{2}\varrho w''^2 + k,$$

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1) Although far from the truth at these high pressures, this simplification has little influence on the computed penetration.

2) The empirical resistance for motion perpendicular to a flat surface is $0.55\,\varrho w^2$ in water, $0.7\,\varrho w^2$ in air.

which means that the lateral surface of the cylinder is considered all the time in contact with the medium (an artificial assumption, like many others, but serving our purposes well).

Further, using here (rather artificially again) a well known hydrodynamical principle, we assume $\delta \frac{w''^2}{2} = p' - p''$, or

$$\delta w''^2 = \varrho (w'^2 - w''^2), \text{ whence:}$$

$$w'' = w' \sqrt{\frac{\varrho}{\varrho + \delta}} \quad \ldots \ldots \quad (5).$$

Combining (4) and (5) we get:

$$w' = \frac{w}{1 + \frac{H}{R}\sqrt{\frac{\varrho}{\varrho + \delta}}} \quad \ldots \ldots \quad (6).$$

The schematic equation of motion we may write thus:

$$\pi R^2 p' = - m \frac{dw}{dt} = - m \frac{d^2x}{dt^2},$$

where $m = 2\pi R_0^3 \delta$ is the mass of the projectile; or, also,

$$\frac{d^2x}{dt^2} = \frac{dw}{dt} = - \frac{1}{R_0}\left(\frac{1}{4}\frac{\varrho}{\delta} w'^2 + \frac{k}{2\delta}\right)\left(\frac{R}{R_0}\right)^2 \quad \ldots \ldots \quad (7).$$

Table I.

Maximum Penetration *(x_m)*, and Final Radius *(R_m)*, in units of Initial Radius ($R_0 = 1$) of a Cylindrical Projectile *($H_0 = 2R_0$)*, of Initial Velocity w_0 (km/sec), moving parallel to its axis:

case a), iron projectile impact into stone, $\delta : \varrho = 3$, $k = 2 \times 10^9$ dyne/cm^2; 1)
case b), stone projectile impact into stone, $\delta = \varrho$, $k = 2 \times 10^9$ dyne/cm^2; 2)
case c), iron projectile impact into iron, $\delta = \varrho$, $k = 2 \times 10^{10}$ dyne/cm^2; 2)
case d), stone projectile impact into iron, $\delta : \varrho = \frac{1}{3}$, $k = 2 \times 10^{10}$ dyne/cm^2. 2)

	case a) $w_0 = 60$	case a) $w_0 = 20$	case b) $w_0 = 60$	case c) $w_0 = 60$	case d) $w_0 = 60$
x_m	7.923	7.351	4.536	4.255	2.785
R_m	4.022	3.636	3.456	3.254	2.504
$2R_m : x_m$	1.02	0.99	1.52	1.54	1.80

1) Limit of fluidity of iron at $w' = 1.18$ km/sec; below that limit $R = R_m =$ const. was assumed.

2) No limit of fluidity of projectile assumed.

Equations (7), (6), (5), (2), and (1) form a system which may be numerically integrated. We notice that geometrical similarity holds (for constant velocity, $t \sim x \sim R_0$). The numerical integration was made by comparatively large steps, thus not of very high accuracy; the accuracy is however quite sufficient for the purpose of our "analytical estimates". The final results of the computation are collected in Table I. The density of stone (one-third of iron) assumed in the computations corresponds to terrestrial rocks, and is lower than the mean density of stone meteorites.

Details of a sample computation are given in Table II.

Table II.

Sample Computation of Iron Meteor Impact into Stone (case a), $w_0 = 60 \times 10^5$ cm/sec, $\delta = 7.8$, $\varrho = 2.6$, $k = 2 \times 10^9$, $R_0 = 1$ cm.

$t \times 10^7$ sec.	x cm	w cm/sec	w' cm/sec	R cm	$-\frac{dw}{dt}$
0	0.000	60.0×10^5	30.0×10^5	1.000	7.5×10^{11}
1	0.596	59.2	35.7	1.149	1.76×10^{12}
2	1.179	57.4	40.2	1.325	2.36×10^{12}
3	1.741	55.0	43.0	1.522	3.57×10^{12}
4	2.273	51.4	43.3	1.730	4.66×10^{12}
5	2.763	46.7	41.0	1.936	5.24×10^{12}
6	3.204	41.5	37.7	2.130	5.36×10^{12}
7	3.592	36.1	33.5	2.306	4.98×10^{12}
8	3.928	31.1	29.1	2.462	4.27×10^{12}
9	4.218	26.8	25.3	2.593	3.58×10^{12}
10	4.468	23.2	22.1	2.711	2.98×10^{12}
11	4.685	20.2	19.2	2.814	2.43×10^{12}
12	4.875	17.8	17.1	2.905	2.05×10^{12}
13	5.043	15.8	15.2	2.986	1.72×10^{12}
14	5.193	14.1	13.6	3.058	1.44×10^{12}
16	5.445	11.2	10.9	3.180	1.00×10^{12}
18	5.649	9.20	8.95	3.279	7.20×10^{11}
20	5.819	7.76	7.58	3.362	5.40×10^{11}
22	5.963	6.68	6.51	3.432	4.67×10^{11}
25	6.142	5.28	5.16	3.519	2.76×10^{11}
29	6.331	4.18	4.10	3.612	1.82×10^{11}
34	6.518	3.27	3.21	3.704	1.19×10^{11}
39	6.667	2.67	2.62	3.778	8.37×10^{10}
47	6.854	2.00	1.97	3.870	5.06×10^{10}
57	7.029	1.49	1.47×10^5	3.956	3.02×10^{10}
67	7.163	1.19	$= w$	4.022	2.02×10^{10}
77	7.272	0.99	$= w$	4.022	1.54×10^{10}
87	7.363	0.84	$= w$	4.022	1.16×10^{10}
107	7.507	0.61	$= w$	4.022	7.1×10^9
127	7.615	0.47×10^5	$= w$	4.022	5.0×10^9
261	7.923	0.00	$= w$	4.022	2.1×10^9

Velocity, within the range of probable meteor velocities, has little influence upon the depth of penetration; the first two cases of Table I may serve as an illustration of this fact.

In our model, the final flattening of the projectile is enormous; thus for case a), $w_0 = 60$, the final shape of the projectile is a flat circular sheet of diameter about 8 R_0 and thickness 1/8 R_0 or 1/64-th of the diameter of the sheet. In connection with that it appears highly significant that the crater No. 4 of the Kaali meteorite group in Estonia, explored by Reinvaldt[1]), shows a flat depression at the bottom. The depression is $2R_m = 5.5$ metres in diameter (diameter of crater 20 metres), and the probable depth of penetration $x_m = 3.5$ metres (counting with the actual soil layer, and assuming vertical incidence); this gives $2R_m : x_m = 1.57$, or a value within the order of magnitude of the data of Table I [suggesting perhaps case b), or the impact of a stone meteorite]. A certain dead funnel, near the centre of the flat depression, ascribed by Reinvaldt to the print of the meteorite (whose dimensions in this case would have been improbably small), may be due to the impact of an iron kernel forming part of the larger stone.

c. Mass affected.

The amount of material crushed and dislocated by a given impact may be estimated in the following, very approximate way. A projectile that moves much faster than the velocity of sound of the medium forces the material of the medium to give way sideways; in this manner a shock wave is formed, consisting of a shell of matter with the trajectory of the projectile as axis moving radially outward with a certain effective velocity; at the margin of the space swept by the projectile this velocity may be set equal to $w''' = \frac{w'}{\sqrt{2}}$ (notations and methods of preceding section[2]). The total "radial momentum" of the shock wave is thus equal to

$$Q = \int_0^{x_m} \frac{\pi R^2 \varrho w'}{\sqrt{2}}\, dx \quad . \quad . \quad . \quad . \quad . \quad (8).$$

1) J. A. Reinvaldt, Kaali Järv — The Meteorite Craters on the Island of Ösel (Estonia), Publ. of the Geological Inst. Tartu, No. **30**, 1933; also, Publ. Geol. Inst. Tartu, No. **11**, 1928.

2) The expression for w''' is got from (5) by setting $\delta = \varrho$ (the medium of uniform density being considered in the present case).

With practical incompressibility of the medium, the shock wave represents chiefly a sort of transfer of radial momentum over a continually increasing volume, with only a feeble transfer of matter; as the volume affected increases, the average velocity of the outward movement, v (= velocity of vibration, not of propagation), decreases in such a manner that the radial momentum will remain constant; thus

$$Mv = Q \quad . \ . \ . \ . \ . \quad (9),$$

where M is the total mass affected at a certain phase. By setting v equal to the "velocity of destruction", at which the kinetic energy of the unit mass is equal to the energy spent in crushing, lifting and ejecting the unit mass of the material, from (9) and (8) it is possible to calculate the mass affected, M, in terms of the mass of the projectile, μ. For the model considered in the preceding section we have $\mu = 2\pi\delta R_0^3$, and

$$\frac{M}{\mu} = \frac{\varrho}{2\sqrt{2v\delta}} \int\limits_0^{x_m} \frac{R^2 w'}{R_0^3}\, dx \quad . \ . \ . \ . \ . \ . \ . \quad (10).$$

The value of the integral in (10) is found by numerical integration equal to 76.98×10^6 [1]) for $w_0 = 6 \times 10^6$ and $\frac{\delta}{\varrho} = 3$ (see Table II); for other cases the integral is very closely proportional to $\frac{\delta}{\varrho} w_0$. Thus, for an impact of any sort of projectile into rock we have, by (10), independently of δ and ϱ,

$$\frac{M}{\mu} = \frac{\varrho}{2\sqrt{2v\delta}}\, 76{,}98 \times 10^6 \left(\frac{w_0}{6 \times 10^6}\right) \cdot \frac{\left(\frac{\delta}{\varrho}\right)}{3} = \frac{76{,}98 \times 10^6}{6\sqrt{2v}} \left(\frac{w_0}{6 \times 10^6}\right), \text{ or with}$$

$$v = 1.55 \times 10^4 \text{ cm sec}^{2)}, \quad \frac{M}{\mu} = 588. \left(\frac{w_0}{6 \times 10^6}\right) \quad . \ . \ . \quad (11).$$

In the case of impact into iron, the energy of rupture is about 9×10^7 erg/gr, or about fifteen times larger than the crushing energy of rock; we may assume that, in the case of

1) This value corresponds to a total radial momentum equal to $\frac{3}{2}\,\mu w_0$, or $\frac{3}{2}$ times the momentum of the projectile.

2) This corresponds to an average mechanical work of 1.2×10^8 erg/gr, as assumed in Section a).

impact, the total mechanical energy spent in crushing and dislocating the material is larger in the same proportion, or that v is about four times the value for rock. Thus, for the impact of a cosmic projectile into iron we get:

$$\frac{M}{\mu} = 147. \left(\frac{w_0}{6 \times 10^6}\right) \quad \ldots\ldots \quad (12).$$

d. Mass of Arizona Meteorite.

The methods developed in Sections *b* and *c* furnish two independent criteria for estimating the probable mass of a projectile able to produce a given meteor crater. Method *b* is based on the observed penetration, x_m, which in connection with Table I leads to an estimate of R_0, and hence of the original mass, μ; method *c* is based on the observed amount of crushed and ejected material. For the Canyon Diablo crater we assume: $x_m = 340$ metres; $\frac{\delta}{\varrho} = 3$ (iron into stone), thus case a) of Table I; $v = 1.55 \times 10^4$ cm/sec, or formula (11); $M = 1.0 \times 10^{15}$ gr.

The different estimates of mass calculated with these numerical data are given in Table III.

Table III.

Estimates of Mass (in tons) of Canyon Diablo Crater Meteorite.

Assumed velocity w_0, km/sec	a minimum from mech. work	b probable mass from penetration	c probable mass from volume of crater
20	60 000	4.8×10^6	5.1×10^6
60	7 000	3.9×10^6	1.7×10^6

Methods *b* and *c* yield identical values of the mass at a velocity of 25 km/sec, which result, however, need not be taken too seriously. The important fact is that both methods give at meteor velocities results of closely the same order of magnitude. It seems safe to conclude that the total mass of the Arizona meteor may have been from two to five million tons.

On the other hand, there is little chance of expecting all that mass to be found somewhere in one large piece. As follows from the theory of section *b*, at the final moment of the first phase of the impact (moment of deepest penetration), all this enormous mass of iron must have been flattened out over an area (only about 100 metres eccentrically placed relative to the rim) of about 330 metres diameter, with an average thickness of from five to seven metres; without doubt all this mass could not have kept together as a single piece, but must have been broken up into thousands of smaller fragments. Further, the moment of deepest penetration hardly represents the final phase of the collision; subsequent movements, perhaps even an explosion due to heat action, may have dislocated the meteor fragments and mixed them with the other debris of the crater.

Tartu, May 1935.

II. The possible Consequences of the Collisions of Meteors in Space.

Abstract.

Like the two preceding papers of this Series[1]), the present, third paper must be regarded as preparatory to a general theory of meteor phenomena, yielding at the same time results of independent significance.

Impacts from numerous small meteors must "eat up" and pulverize, in the course of time, meteoric material. In this grinding action meteors belonging to interstellar space must be much more efficient than those belonging to the solar system.

Numerical estimates indicate that during 3000 million years most stone meteors of original radius about 0.1 cm, and most iron meteors of original radius 0.01 cm, or smaller, must have been destroyed by collisions.

Assuming that Leonids and Perseids are chiefly stones (spectroscopic evidence), the peculiarities of their "luminosity curves", i. e. the absence of faint members, if ascribed to the effect of collisions, give for these streams lower and upper limits of age 500 and 7000 million years respectively, in satisfactory agreement with the supposed age of the solar system.

As the result of grinding by collisions, the ratio of the surface of the meteor to its mass increases steadily; leaf-shaped bodies of a surface rich in cavities (small craters) are formed, which, when penetrating into the terrestrial atmosphere, are vapourized at greater heights than would be a compact (spherical) body of same mass and velocity.

As the result of collisions, most meteors are likely to have acquired considerable rotation, of the order of 30 m/sec "equatorial" velocity for average "naked-eye" objects.

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1) A, and I, cf. p. 3.

Collisions between meteors of large relative velocity lead to the formation of smaller and smaller fragments, which process in the course of time may end in complete volatilization. The importance of this process we are going to consider below. The data of Paper I are of great use for our purposes. The quantitative results aim at the order of magnitude only, but, even as such, they are eloquent enough to point out a number of important problems in meteor research.

The surface of a meteor travelling in space is subject to the impacts of other meteors. The majority of collisions must take place with particles which are small as compared with the meteor itself; collisions with larger bodies must be comparatively rare because the large meteors are known to be less numerous than the small meteors. Each impact of a small particle produces on the surface of the meteor a sort of a small "meteor crater"; the dust-like fragments of meteoric material, very small as compared with the meteor, and even with the projectile itself, will be dispersed in space by the shock; thus the meteor loses all the "mass affected" [1]), which partly is volatilized, but mostly, however, is converted into solid dust of "ultra-telescopic" meteors of a size that is practically outside the range of our observations. Except when molecular dimensions are approached, geometrical similarity may be assumed to hold, and the ratio of mass lost to the mass of the projectile may be taken as defined by (11) and (12) of Paper I.

a. Assumptions.

A fundamental difference with respect to the efficiency of collisions must exist between the solar meteors (members of the solar system), and the hyperbolic meteors (supposed to be mostly members of the galactic system). The solar meteors, being in many cases connected with comets, are known to move mostly

1) Cf. Paper I.

along very elongated ellipses. A meteor moving along such an orbit spends most of its time near aphelion, where the velocity and space density (number of meteors per unit volume) are much smaller than the same observed by the terrestrial observer. Collisions of solar meteors with solar ones must therefore be comparatively rare. On the other hand, the velocity and space density of hyperbolic meteors is only moderately influenced by the solar gravitational field, so that these quantities as observed by the terrestrial observer are at least of the same order of magnitude as in interstellar space. The number of hyperbolic meteors seen by the terrestrial observer being comparable with the number of solar meteors[1]), at aphelion distances of most solar meteors, the hyperbolic meteors may considerably outnumber the solar meteors. Most meteor collisions must therefore take place with hyperbolic meteors.

In the following we try to make a minimum estimate of the effect of collisions. For solar meteors therefore we assume aphelion conditions as the least favourable for collisions, and we neglect collisions with other solar meteors; we consider only collisions produced by hyperbolic meteors.

For the sake of simplicity, we assume the masses of meteors classified according to stellar magnitudes[2]), and the frequency of magnitudes, or masses, to be a geometrical progression with a ratio of increment of 2.512 per magnitude (such a ratio is of the right order for telescopic meteors up to about the 10-th zenithal magnitude, the limit of definite information). As minimum frequency of hyperbolic meteors at great distances from the sun we assume one-tenth of the average frequency of all meteors observed by the terrestrial observer. This minimum frequency may be estimated at 0.2 meteors per hour of the second zenithal magnitude (average mass 10 milligram at velocity 60 km/sec) per 30 000 km^2 of a horizontal surface, or 5.84×10^{-12} per cm^2 and year, with a corresponding increment of the number for each fainter magnitude class. The surface of the meteor is, from the standpoint of cosmic impacts, "horizontal" in the same geometrical sense as we call horizontal the boundary of our atmosphere, and the estimated frequency of meteor encounters

1) E. Öpik, On the Distribution of Heliocentric Velocities of Meteors, Harvard Circ. No. **391**, 1934.

2) Reckoning by integer magnitudes only.

applies thus directly to the boundary of the meteor (except when concave), which we assume spherical, for the sake of simplicity.

We consider first solar meteors composed of stone. The average relative velocity of the projectiles we set equal to 60 km/sec. Without regard to the composition of the impinging projectiles, by formula (11) of Paper I the mass lost by the meteor may be assumed equal to 588 times the mass of the projectile. The impact of a 2-nd magnitude meteor pulverizes thus $0.010 \times 588 = 5.88$ gram of the meteor substance. With a density $\varrho = 3.4$ this means an average loss per year of a layer of

$$\frac{5.88}{3.4} \times 5.84 \times 10^{-12} = 1.01 \times 10^{-11} \text{ cm} \quad . \; . \; . \; . \; . \quad (a),$$

due to impacts of 2-nd zen. mag. meteors only. For any other magnitude class of the impinging meteors, with the law of the frequency of masses assumed, the total mass, and the total effect will be the same. Let n denote the effective number of magnitude classes of the hyperbolic meteors which are active in grinding the surface of a meteor of a given radius R; we have the decrement of radius of such a stone meteor:

$$\frac{dR}{dt} = -1.01 \times 10^{-11} \, n \quad . \; . \; . \; . \; . \; . \quad \left(\frac{\text{cm}}{\text{year}}\right) \quad . \; . \; . \; . \; . \; . \quad (1).$$

Formula (1) applies chiefly to impacts of relatively small projectiles. Projectiles which are larger than $\frac{1}{588}$ th of the mass of the meteor destroy less than supposed above: collisions with such projectiles may destroy all the meteor, but not more. Also, collisions with large projectiles are comparatively rare. We disregard the collisions with projectiles larger than the above-mentioned limiting relative size, considering in (1) only magnitude classes of the impinging meteors which are at least seven magnitudes less massive[1]) than the given meteor. In this way we again underestimate the destructive action of the collisions. In this case n is determined by the effective lower limit of meteor masses. This lower limit is rather indefinite. We may set two extreme values for the lower limit of meteor masses:

1) the steady increase of meteor numbers with decreasing luminosity, and mass, has been definitely observed up to about

1) We remind that here magnitudes are used to measure masses, not uminosities.

the tenth magnitude; fainter telescopic meteors have been observed occasionally; it appears safe to assume zenithal magnitude 13 as a directly observed upper limit of minimum mass;

2) the total mass of meteors in a given volume of space must be finite; also, the size of particles cannot decrease indefinitely; the minimum possible size of individual particles is represented by molecules, or atoms (not counting electrons); collisions produced by single molecules are unable to kick out from a solid surface more than a small multiple of their own mass, because the forces of chemical affinity (molecular forces) present too great an obstacle to destruction; thus, very small particles, approaching the dimensions of molecules, are rather inefficient in destroying by impacts a solid body; from these considerations, we get the maximum amount of the active impinging mass by setting the lower limit of masses in a steady increment of 2.512 : 1 equal to limiting zenithal magnitude about 52 (mass of molecule); it seems that by extrapolating the frequency of masses over such an enormous range we obtain a large overestimate.

Thus, for stone meteorites of magnitude class m, we have two limits for the possible value of the "coefficient of destruction", n:

$$52-(m+7)>n>13-(m+7), \text{ or}$$

$$45-m>n>6-m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

We notice that in our system of measuring masses, $m=2$-nd zen. mag., or mass 10 milligram[1]), corresponds very closely to $R=0.1$ cm (stone), or that

$$m=-5.5-7.5 \log R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

the radius being given in cm.

For an average naked-eye meteor, $m=2$, $R=0.1$ cm,

$$43>n>4, \text{ and from (1):}$$

$$4\times10^{-10}>-\frac{dR}{dt}>4\times10^{-11} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Assuming $t=3\times10^{9}$ (minimum age of the universe), we get the total decrease of radius

$$1.2>-\Delta R>0.12 \text{ cm},$$

1) Cf. Paper A, p. 39.

or values larger than the meteor itself. In other words, in present conditions, a comparatively rapid disintegration of "naked-eye" stone meteors must take place. Assuming the minimum rate of disintegration, we find that an average stone meteor which at present is of the 2-nd zen. magnitude, or $R = 0.1$ cm, must have been originally (3×10^9 years before our era) of an average radius 0.22 cm, or of average magnitude — 0.5, which means an average decrease of mass in the ratio 10:1. Meteors of original radius 0.1 cm must have disappeared altogether.

Iron meteors must be more persistent. From (12) of Paper I, the destructive action of collisions upon iron meteors is estimated at one-quarter (by mass) of the corresponding action upon stone meteors, or about one-ninth by volume: thus we may set $\frac{dR}{dt}$ for iron at nine times smaller than for stone. This makes, for $t = 3 \times 10^9$ years,

$$0.13 > -\Delta R > 0.013 \text{ cm};$$

naked-eye iron meteors may be reasonably persistent for periods of time comparable with the supposed age of the universe; rapid disintegration may take place among telescopic iron meteors. Assuming the minimum rate of disintegration as given above, we find that iron meteors which at present are of the 9-th zen. magnitude may have decreased in mass in the ratio 10:1 during 3×10^9 years. Fainter telescopic meteors must have disappeared during such an interval of time. We notice that our estimates refer to hyperbolic meteors equally well, unless these move more or less parallel in space with identical velocity so that collisions cannot occur frequently enough: rejecting this latter possibility as an improbable one, we may account for the existence of faint telescopic meteors only by the assumption that their supply is continually renewed: 1) either through fragments from collisions of larger meteors, or 2) from condensation of interstellar gas. In other words, faint telescopic iron meteors, and most of the fainter naked-eye stone meteors cannot be as old as the universe is.

b. Age of Leonids and Perseids.

These two meteor streams seem to be conspicuous by the practical absence of faint telescopic members. As a drastic case may be mentioned the splendid Leonid display as observed on Nov. 16—17, 1931, at Flagstaff, Arizona; while each of the visual observers, including the writer, succeeded in tracing on maps about 100, mostly bright, Leonids, prof. S. L. Boothroyd observing with a four inch telescope recorded only seven meteors, none of which could be a Leonid. The magnitude — frequency curves for these two streams, instead of showing a steady increment, seem to bend over, yielding maxima at places which may be set at $R = 0.10$ cm for Leonids (at about 1-st appar. mag., $w = 71$ km/sec), and at $R = 0.06$ cm for Perseids[1]). Assuming an original distribution with a steady increment (2.512 : 1 in the present case) over all magnitudes, thus without a maximum in the frequency curve, assuming that the average final radius is

$$R = R_0 - \Delta R,\ \Delta R = \text{const.} = -\frac{dR}{dt} \times t,$$

a simple mathematical analysis indicates that the final frequency of magnitudes will show a maximum when $R = {}^1/_4 R_0$, or $\Delta R = 3R$. This gives $\Delta R = 0.30$ cm for Leonids, and 0.18 cm for Perseids. The theoretical frequency decreases very rapidly for magnitudes fainter than the magnitude of maximum frequency, and the final frequency shows much resemblance to the observed frequencies of the magnitudes of the meteor streams under examination.

In connection with this, we notice that all the eight spectra of Leonids[2]) hitherto observed belong certainly to stones (H and K lines of Ca^+ prominent). This indicates that stones are the predominant constituent of the Leonid shower. For Perseids the data are less numerous, but apparently they also are mostly stone meteors. On the other hand, of the fourteen spectra of "sporadic" meteors (mostly hyperbolic?), only five are certainly stone, whereas nine or 64 per cent are apparently iron[2]). These data refer to meteors whose spectra could be photographed, i. e. to comparatively bright fireballs, of large radius for which the process of disintegration considered above must have been unimportant. Therefore these figures give us an idea of the ori-

1) These estimates of the radius are rather insensitive to the precise magnitude of the maximum.

2) P. M. Millman, An Analysis of Meteor Spectra, 2nd paper, Harvard Annals **82**, No. 7, 1935.

ginal relative frequency of stone and iron, unaffected by disintegration. It appears safe to assume that the Leonids and Perseids are predominantly stone, which from the above considerations appears to be subject to rapid disintegration. With the rate of disintegration as given by (4), and the values of ΔR as estimated above, we find the age of the Leonids from 7.5×10^8 to 7.5×10^9 years, and the age of the Perseids from 4.5×10^8 to 4.5×10^9 years. Without laying too much stress on the actual figures obtained, because they depend upon so many assumptions, we notice that the figures are of the order of magnitude of the supposed age of the solar system (and, probably, also of the universe itself)[1].

Before concluding, it might be safe to inquire into other possible reasons for the lack of small meteors in the periodic meteor showers discussed here. One of the possibilities is the lack of these small particles "from the very beginning"; we cannot disprove such a hypothesis, although it seems rather improbable. Another cause which might have led to an apparent lack of small meteors is radiation pressure; changing the orbit, radiation pressure might have forced the smaller particles to move along paths which do not intersect with the earth's orbit; in this way a kind of "sedimentation" takes place in the meteor stream. It is easy to show, however, that the actual effect is too small to be observed. The radiation pressure is equivalent to a change of the gravitational constant; for absorbing spheres that are large compared with the wave length and are moving in the solar gravitational field, the relative change of the gravitational constant is given by (5) and (5′):

$$\Delta g = -\frac{1.78 \times 10^{-5}}{R} \text{ (Stone)} \quad . \; . \; . \; . \; . \; . \quad (5), \text{ and}$$

$$\Delta g = -\frac{0.77 \times 10^{-5}}{R} \text{ (Iron)} \quad . \; . \; . \; . \; . \; . \quad (5').$$

Meteor streams are mostly observed near their perihelia. In such a case, the maximum displacement of the orbital stream line we obtain by assuming that the small and the large ($R = \infty$) particle start at aphelion with equal velocities. From element-

1) E. Öpik, Meteorites and the Age of the Universe, Popular Astronomy **41**, No. 2, 1933; Harvard Reprint No. **84**.

ary considerations we find the change of the perihelion distance q as follows:

$$\Delta q = -q \Delta g \frac{q+r}{r} \quad . \; . \; . \; . \; . \quad (6),$$

where r is the aphelion distance. For observable meteor streams $q \leqslant 1$, and r is large as compared with q; $\frac{q+r}{r}$ is very close to 1. Setting $q = 1$ in (6), we actually overestimate the effect.

Thus $\Delta q = -\Delta g$. For $R = 0.06$ cm (Perseids of the 4-th magnitude), according to (5),

$$\Delta q = +0.0003 = 45\,000 \text{ kilometres},$$

or the distance travelled by the earth in twenty-five minutes. On the other hand, the Perseids and many other meteor streams are known to be spread over elliptical rings of a thickness at least fifty times larger than that, by causes other than radiation pressure; thus, the effect of radiation pressure upon naked-eye meteors must be negligible, being entirely lost in a number of other effects[1]) which partly are known (perturbations, which equally affect large and small bodies). The radiation pressure may become important, from the standpoint of "sedimentation", only from about the 15-th mag. There seems to be no escape from the conclusion that the observed absence of faint members of meteor showers is due to the effect of collisions with meteoric particles of interstellar space. Numerical estimates indicate that the time of the "unshielded" existence of the Leonid and the Perseid showers may be equal or close to the supposed age of the solar system. This conclusion does not depend upon possible changes in the shape of the orbits of these streams during the interval of time mentioned above, supposing that the aphelion distances of the orbits remained large (> 4 astr. units) during all the time of the existence of the shower.

c. Shape of meteors.

As another consequence of the continual grinding of the surface of meteors by collisions there follows a curious system-

1) The momenta contributed by consecutive collisions with small hyperbolic meteors may play the chief rôle; this explains why large masses are relatively more abundant on the days of the maxima of the Perseids (and Leonids). (Cf. Tartu Observatory Publ. **XXV**, 1 and 4, 1922, 1923.)

atic change in their shape. The original shape of a meteor, although often assumed to be spherical for purposes of schematization, is certainly in most cases far from being such; whether fragments of larger bodies, or products of crystallization in space, there can be hardly any doubt that the original shape must have been as irregular as we know terrestrial fragments are. Most of the grinding is produced by a great number of small collisions, the effect of which is a more or less uniform decrease of the dimensions in all directions by an amount ΔR; let the major and the minor original diameters of the fragment be a_0 and b_0; the final probable diameters will be

$$a = a_0 - 2\Delta R, \text{ and}$$
$$b = b_0 - 2\Delta R.$$

Evidently

$$\frac{a}{b} > \frac{a_0}{b_0},$$

or a small original inequality of the diameters is intensified through the effect of collisions. For example, let us take a prism of original dimensions 0.60, 0.80, and 1.00 cm respectively, which proportions may be regarded as typical for an average fragment, and let $\Delta R = 0.24$ cm (mean estimated for Leonid and Perseid stones); the final probable dimensions are 0.12, 0.32, and 0.52 cm, or the prism becomes more elongated, and still more flattened. Of course, some major collisions may easily break into pieces such a flattened figure, thus preventing it from becoming a very thin leaf; but a general tendency for producing flattened and elongated fragments must exist. Together with the traces of major collisions in the form of cavities, branches, holes, the effect will consist in increasing the relative surface of the meteors. From estimates made above, we may expect that naked-eye stone meteors possess a much greater surface per unit of volume than iron meteors. From the standpoint of the meteor theory (process of vaporization in the terrestrial atmosphere), this circumstance seems to be of considerable importance.

d. Rotation.

From the preceding it appears that a meteor being subject to collisions which may destroy it or break off fragments from it, must acquire relatively rapid rotation as another consequence of the collisions. Possible rotation has a great influence upon the theory of meteor phenomena in the terrestrial atmosphere; therefore we may try to evaluate the order of magnitude of the minimum rotational velocity of meteors.

The problem is rather complicated because the mass of the meteor decreases while it acquires rotational momentum through collisions. Certain simplifications are necessary.

In an inelastic collision, part of the relative momentum is spent in producing rotation. When the impinging mass m_1 is small as compared with the mass m_2 of the meteor — which is almost always our case — the average "equatorial" velocity of rotation acquired through the collision of relative velocity w may be estimated at

$$u_e = {}^1/_2 \frac{m_1}{m_2} w \quad ^{1)} \quad . \; . \; . \; . \; . \quad (7).$$

There is a limitation to the formula set by the rigidity of the material; a momentum of rotation larger than a certain limit cannot be intercepted; above the limit the material yields and the meteor breaks, in other words it does not behave any more like a single rigid body. An obvious necessary condition for the complete absorption of the momentum of the projectile is that the latter must not penetrate through the body of the meteor. Another condition is that there must be left something to rotate — a considerable fraction of the meteor must remain in one solid block. As an upper limit for complete absorption of the momentum of the impact we assume collisions which destroy not more than one-quarter of the mass of the meteor. From Paper I it appears that the condition of non-penetration is well fulfilled in this case (the crater formed by impact is shallow). In the following we consider only collisions which destroy at once not more than one-quarter of the meteor. By disregarding initial rotation, as well as rotation produced by "major collisions" i. e. those which destroy more than one-quarter of the original mass, we evidently get a minimum estimate of the total rotational effect. On the other hand, with the limitation set above, in assuming constant mass and dimensions of the meteor during all its life time we must get the right order

1) Theory gives a somewhat larger value for a rigid sphere.

of magnitude of the rotational velocity. Disregarding the continuous shrinking of the meteor during its life time or assuming its size to be equal to the final present size actually means a slight overestimate of the total rotational effect [because the effect is stronger for small meteors, cf. formula (9)]. Discarding the major collisions means underestimating the rotational effect. Thus the tendency of the two simplifications is to cancel the errors introduced by them.

Let us classify, as before, the masses of meteor projectiles by whole magnitude classes. Let the largest class which we take into account yield altogether K impacts, each of which is capable of producing an average rotational effect u_e ("equatorial" velocity of rotation); the rotational impulses are of an accidental character, and their summation therefore may be assumed to follow the quadratic rule; the total average rotation produced by the given magnitude class is thus $u_e \sqrt{K}$ (when $K > 1$). The next magnitude class of projectiles produces impulses which are $\frac{1}{a}$ times smaller, and a times more numerous (with the adopted law of distribution of meteor masses), where $a = 2.512$; thus the total impulse from this class is $\frac{u_e}{a}\sqrt{aK} = \frac{u_e\sqrt{K}}{\sqrt{a}}$. Evidently the rotational impulse produced by all magnitude classes is

$$U_e = u_e \sqrt{K}\sqrt{1+\frac{1}{a}+\frac{1}{a^2}+\ldots} \qquad (8).$$

The sum under the square root is a rapidly converging series and depends very little upon the number of members. Four members give 1.62, and an infinite number of members gives 1.66. Thus the uncertainty in the distribution of non-observable, ultra-telescopic meteors does not influence our estimates in this case. We set, as a minimum value of the mean equatorial velocity acquired,

$$U_e = 1.2\, u_e \sqrt{K} \qquad (8').$$

We see that a few impacts of large projectiles practically determine the velocity of rotation.

Taking a typical stone meteor of $R = 0.1$ cm (2-nd zen. magnitude at $w = 60$ km/sec), and an average velocity of 60 km/sec, we find that the largest projectile which must be taken into account, i. e. which is capable of destroying one-quarter of the meteor, has a mass ratio $\frac{m_1}{m_2} = \frac{1}{4.588} = \frac{1}{2350}$ (according to form. (11) of Paper I). The spherical surface of $R = 0.1$ cm is $0.13\ \text{cm}^2$; with the frequency of meteors assumed in Section a) of the present paper, we find that during 3×10^9 years the probable number of collisions with such particles is

$$K = 5{,}84 \cdot 10^{-12} \cdot 3 \cdot 10^9 \cdot 0{,}13 \cdot 2350 = 5{,}3.$$

The average velocity of rotation of a 2-nd mag. stone is thus, according to (8) and (7),

$$U_e = 1{,}2 \cdot \frac{1}{2} \cdot \frac{1}{2350} \cdot 6 \cdot 10^6 \sqrt{5.3} = 3600\ \text{cm/sec.} \text{ (for stone, } R = 0.1 \text{ cm).}$$

We have disregarded here the additional impulse from the ejection of the debris produced by the impact; the ejection of fragments or vapours more or less at right angles to the surface adds to the rotational impulse of the projectile in the case of a non-spherical body; the addition may amount to from 30 to 70 per cent, according to the considerations put forward in Paper I. In disregarding the additional impulse, we remain on the safe side of the intended minimum estimate. The resulting probable velocity of rotation is nevertheless considerable, corresponding to 6000 revolutions per second.

The formula for the combined effect of rotation tacitly assumed that the mean rotation, already acquired, is small as compared with the "statistical equilibrium velocity", $w\sqrt{\frac{m_1}{m_2}}$. For $w = 6.10^6$ we find the equilibrium velocity equal to the deduced value of U_e for $\frac{m_1}{m_2} = 3{,}6.10^{-7}$, or for projectiles of the eighteenth magnitude, eight magnitudes fainter than the largest projectiles taken into account; projectiles smaller than this limit will diminish instead of increase the average rotational velocity; however, in formula (8) the influence of these fainter members is negligible and need not be taken into account. Small meteor particles, up to molecules and atoms, by their impacts will tend to slow down the acquired rotation, creating a sort of viscosity of the medium. The retardation, however, is considerable only when the mass which comes into contact with the surface of the meteor is considerable as compared with the mass of the meteor. The maximum estimate of the amount of meteoric matter in Section a) of Paper II indicates that retardation (in 3.10^9 years) becomes important only for meteors of $R < 0.006$ cm, or from about the eleventh zenithal magnitude on. If ϱ is the combined density of meteoric matter (dust) and gas in space, the total mass which is swept by a meteor in 3.10^9 years with velocity 60 km/sec is equivalent to a stone layer of $4.10^{22}\,\varrho$ cm depth. Estimates of the density of interstellar matter [$\varrho \sim 10^{-26}$ [1])] indicate that the mass is equivalent to a layer ~ 0.0004 cm, or that only meteors of the order of $R = 0.001$ cm may be affected. The conclusion is that only faint telescopic meteors may have experienced the influence of retardation, whereas naked-eye meteors are expected to maintain the considerable rotational momentum produced by collisions with relatively large particles.

For other sizes of meteors, the surface is proportional to R^2, whereas the frequency of the projectiles of maximum size (in constant ratio to the mass of the meteor) varies as $\frac{1}{R^3}$ (with the adopted law of the frequency of meteor masses); the number of impacts of such projectiles thus varies as

$K \sim \frac{1}{R}$. Hence, by (8′),

$$U_e = 3600 \sqrt{\frac{0.1}{R}} \ \frac{\text{cm}}{\text{sec}} \quad \ldots\ldots \quad \text{(Stone) (9)},$$

$$\text{when } K = \frac{5{,}3.0{,}1}{R} = \frac{0.53}{R} > 1,$$

or for $R < 0.53$ cm. Thus, formula (9) is more or less valid for stone meteors fainter than zenithal magnitude — 3 (velocity 60 km/sec). On the other hand, there exists an upper limit of rotational velocity above which the body breaks from centrifugal force. For stone this upper limit may be set at $U_e \leqslant 1{,}1.10^4$ cm/sec, for iron at $U_e \leqslant 5{,}5.10^4$ cm/sec; the estimate depends upon the shape, for a given shape upon the ratio of tensile strength to density.

Thus, on the side of faint meteors, formula (9) is valid up to about $R = 0.01$ cm, or to about the 8-th zen. magnitude; below this limit, the rotational velocity may be assumed constant, equal to the maximum limits given above.

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1) Gerasimovič and Struve, Astroph. Journal **69**, 7, 1929.

For large meteors, when $K < 1$, formula (8′) loses its simple meaning; being mathematically valid in the sense that it still gives the quadratic mean velocity of rotation, the formula becomes inconvenient because the frequency of velocities is peculiar in this case: the distribution of rotational velocities is "one-sided", a few large velocities occurring among a great number of small velocities. Nevertheless, when K is not too small ($K > 0.1$), the inconvenience of using the quadratic mean is still insignificant (for our order-of-magnitude computations), whence it follows that our formula (9) applies satisfactorily even for stones of $R = 5$ cm, or up to about zenithal magnitude -10. In other words, the formula applies for all the range of naked-eye meteors.

For iron meteors, the "integrity ratio", $\frac{m_1}{m_2}$ may be assumed four times larger than for stones [Paper I, (12)]; for equal radius, the mass is proportional to density; hence nine times larger projectiles are to be taken into account; their frequency is nine times smaller, or K must be taken smaller in the same ratio; we find that for equal radius, the average (mean quadratic) rotational velocity of iron meteors is $\frac{4}{\sqrt{9}} = \frac{4}{3}$ times larger than for stones or practically the same as for stone meteors of equal radius. Thus, for the order of magnitude, formula (9) remains valid also for iron meteors.

For the mean period of revolution we get, according to (9),

$$P = \frac{2\pi R}{u_e} = \frac{R^{\frac{3}{2}}}{1800} \; . \; . \; . \; . \; . \; (\mathrm{sec}) \; . \; . \; . \; . \; . \; (10).$$

The so-called flickering meteor trails are very likely explained by rotation. For an average photographed meteor we may set $R \sim 1$ cm, whence theoretically $P = 1/1800$ sec. Let us consider for example the flickering trail studied by Olmsted [1]). Assuming the flickers to correspond to rotation, with an apparent angular velocity of the meteor from ten to forty degrees per second, the observed flickers must have had a period from 1/300 to 1/1200 of a second. The figures fall well within the expected order of magnitude.

December, 1935.

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1) Margaret Olmsted, An Unusual Meteor Trail, Harvard Coll. Observ. Bull. No. 888, 16, 1932.