PUBLICATIONS
DE
L'OBSERVATOIRE ASTRONOMIQUE DE L'UNIVERSITÉ DE TARTU
TOME XXVIII № 5

THE *TiO* COLOUR EFFECT, AND THE DENSITIES OF *M* STARS

BY

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TARTU 1936

Acta et Commentationes Universitatis Tartuensis (Dorpatensis) A XXX.$_2$.

Printed by C. Mattiesen Incorp., Tartu 1936.

Abstract.

The influence of *TiO* absorption on the colour of *M* type stars is discussed; the *TiO* correction increases rapidly with the advancing *M* spectrum, reaching the value 3,3 mag. for *M* 9.

The mean "true" colour indices of *M* giants and supergiants are computed; the colour indices of the supergiants are found to be about 0,46 mag. greater than those of the ordinary giants. From the general trend of the colour — absolute magnitude relationship between *GO* and *M*, supported by a few directly observed colours of *M* dwarfs, a constant difference of 0,33 mag. between the colours of dwarfs, and of ordinary giants, is assumed.

The average bolometric absolute magnitudes of *M* stars are calculated from the published visual absolute magnitudes corrected for *TiO* absorption. On the basis of an empirical linear relation of log mass to bolometric magnitude, the typical hypothetical masses of *M* dwarfs are computed. The mean radii and densities of *M* giants, as well as the data for stars having the greatest radii and smallest densities, are computed; radii greater than 300 times the sun's radius appear to be quite numerous.

The computed angular diameters of α Scorpii, α Orionis, and α Herculis are compared with those derived from stellar radiation measurements by Pettit and Nicholson, and with the diameters measured with the interferometer by Michelson and Pease, after correcting the latter for the limb darkening effect. The general agreement is satisfactory, except for the interferometer value of α Herculis. This discrepancy, requiring for its removal apparently a *TiO* correction equal to 80 per cent of the assumed value, is more probably due to the uncertainty in the colour index of this star, influenced by its *F* type companion.

The deviation from black body radiation revealed by the *M* type stars is almost entirely explained by *TiO* absorption.

With trigonometric parallaxes and hypothetical masses as a basis, the individual densities for 97 *M* dwarfs are calculated; the mean densities of these stars are found to be nearly equal to the sun's density over the whole spectral range from *M*0 to *M*5; the cosmic, or true, spread of the densities of *M* dwarfs apparently does not exceed ±17 per cent; this value is remarkably small, indicating the great similarity of the internal structure of *M* dwarfs.

Such a small spread in density appears to be the best evidence of the reliability of our assumptions regarding the *TiO* colour effect, and the constancy of the difference between giant and dwarf colour indices.

The frequency curve of density logarithms of *M* dwarfs shows an excellent coincidence with a Gaussian error curve, representing more likely the error distribution, than the true distribution of density logarithms.

1. The *TiO* Colour Effect.

In a recent paper on The Densities of Visual Binary Stars[1] Öpik and Gabovitš considered the quantitative influence of *TiO* absorption on the colour, and the visual magnitude, of the *M* stars, in order to make the necessary correction to the density of the few *M* binaries found in their material. The following method was used: as the photographic magnitude (m_p) is less influenced by *TiO* absorption than the visual one, we may obtain the true effective temperature (T) of an *M* star from

$$m_p - m_b = \frac{36100}{T} + 10 \log T - 43.40 \quad \ldots\ldots \quad (1),$$

where m_b denotes the bolometric magnitude; the constant is found empirically from *K* stars, for which *TiO* absorption is insignificant. We substitute the temperature calculated in such a manner in the formula given by Öpik[2] for the colour-temperature relation:

$$\frac{C_2}{T} = 1.47\,C + 1.82 \quad \ldots\ldots \quad (2),$$

where C is the colour index in a special system[3], and $C_2 = 14300$; we thus obtain the "true" colour C' of an *M* star. The *TiO* correction is then $\varDelta C = C' - C$, C being the observed colour. From 21 *M* stars of Pettit and Nicholson's[4] list we found:

Table I.

Dependence of *TiO* Correction ($\varDelta C$) upon Spectrum.

Sp.	*M* 0	*M* 1	*M* 2	*M* 3	*M* 4	*M* 5	*M* 6
$\overline{\varDelta C}$	0.14 (3)	0.15 (1)	0.25 (6)	0.48 (3)	0.70 (3)	1.32 (3)	1.47 (2)

The number of stars is given in parentheses. The values of $\varDelta C$ actually used in the former publication were taken from

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[1] J. Gabovitš and E. Öpik, *Publ. Tartu Obs.*, **28.** 3, 1935.

[2] E. Öpik, *Ap. J.*, **81**, 177, 1935.

[3] E. Öpik, *Publ. Tartu Obs.*, **27.** 1, 1929.

[4] E. Pettit and S. Nicholson, *Ap. J.*, **68**, 279, 1928; *Mt. W. Contr.*, No. **369.**

a smoothed curve*. Table II gives these values, extrapolated figures being given in brackets.

Table II.

Assumed Dependence of ΔC upon Spectrum.

Sp.	$M0$	$M1$	$M2$	$M3$	$M4$	$M5$	$M6$	$M7$	$M8$	$M9$
ΔC	0.08	0.14	0.28	0.48	0.79	1.18	1.65	[2.2]	[2.8]	[3.3]

Thus we have to correct the observed colour and the visual absolute magnitude (M_v) of an M star in order to obtain the "true" data, C' and M'_v, as follows:

$$\left.\begin{aligned} C' &= C + \Delta C \\ M'_v &= M_v - \Delta C \end{aligned}\right\} \quad \ldots\ldots \quad (3).$$

In the above mentioned paper[1] the writers considered only $M0$ — $M6$ stars; there was no need to use the extrapolated figures. In order to test the validity of the extrapolated portion of the *TiO* colour correction curve, we added to the material considered by Öpik and Gabovitš a few long period variables of late M type, for which the observed colour is given by Gerasimovič, Shapley, and Miss Cannon[5], and which are present at the same time in Pettit and Nicholson's list. With the method described above we calculated the *TiO* correction for these stars; Table III shows the results.

Table III.

TiO Correction of Colour (ΔC) for Stars of Late M Type.

Star	Sp.	m_p	m_b	T	C'	C	ΔC
R Hya max.	$M6e$	5.6	0.29	2430	2.8	1.1	1.7
X Oph max.	$M6e$	7.6	1.87	2340	2.9	0.8	2.1
X Oph min.	$M7e$	9.7	2.45	2060	3.5	0.9	2.6
R Hya min.	$M9e$	11.0	1.18	1730	4.4	1.5	2.9
o Ceti min.	$M9e$	10.4	0.90	1770	4.3	1.3	3.0

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* [1], Fig. 1.

[5] B. P. Gerasimovič and H. Shapley; Annie J. Cannon; *Harv. Bull.*, **872**, 25, 28, 1930.

In Fig. 1 these individual values of ΔC are compared with our former curve; the agreement of these points with the extrapolated portion of the curve is excellent; the *TiO* correction of colour may now be regarded as well established over all the range of the *M* class.

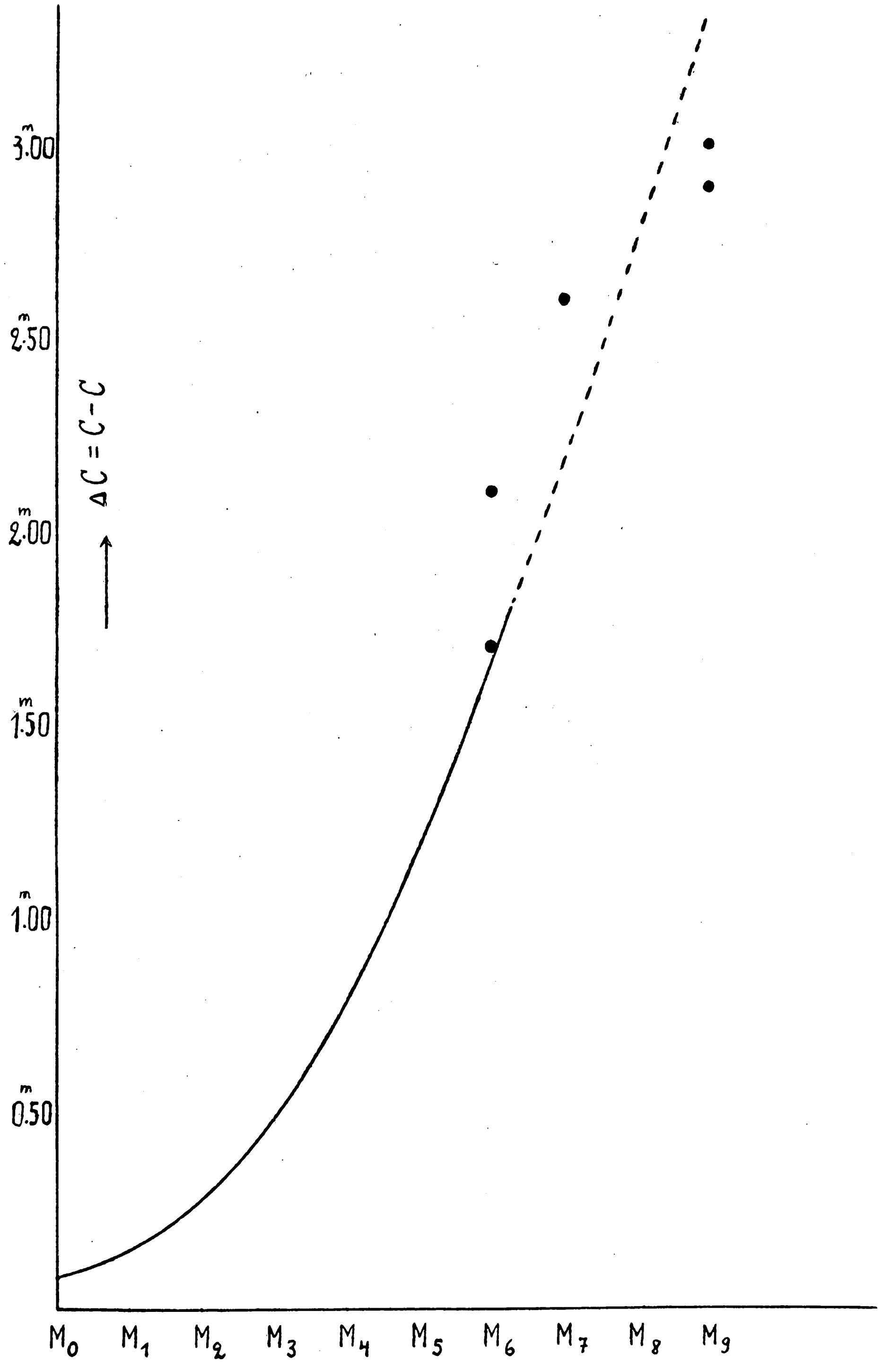

Fig. 1. The *TiO* correction of colour index (ordinates), as function of spectrum (abscissae).

2. The Spectral Systems.

As shown by Fig. 1, the *TiO* correction increases rapidly with the advancing *M* spectrum; a slight error in the spectrum produces a considerable error in the colour, and in the magnitude. Therefore, with the introduction of ΔC in the study of *M* stars, the knowledge of the exact spectral subdivision and the use of an uniform spectral system become of great importance.

In practice we have at our disposal two spectral systems for the *M* stars — the Mount Wilson, and the Victoria systems. From stars observed at both these observatories we derived the correlation between the two systems, as given by Tables IVa and IVb; *n* is the number of stars.

Table IVa.

Victoria Spectrum as Function of Mt. W. Spectrum.

Mt. W.	Vict.	*n*
M 0	*K* 7.6	16
M 1	*M* 0.3	12
M 2	*M* 1.8	25
M 3	*M* 2.9	18
M 4	*M* 5.2	6
M 5	*M* 6.1	7
M 6	*M* 8.0	2

Table IVb.

Mt. W. Spectrum as Function of Victoria Spectrum.

Vict.	Mt. W.	*n*
M 0	*M* 1.2	15
M 1	*M* 1.8	9
M 2	*M* 2.2	16
M 3	*M* 2.6	7
M 4	*M* 3.0	11
M 5	*M* 4.0	5
M 6	*M* 4.5	4
M 7	*M* 5.0	2
M 8	*M* 5.7	3

As shown by Fig. 2, the correlation between the two systems is practically linear. The full line there refers to Table IVa, the dotted one — to Table IVb.

Theoretically there seems to be some advantage in using the Victoria system in our study of *M* stars, because the ΔC-spectrum curve shows in this case a smaller slope than in the case of the Mt. W. spectrum. Nevertheless, in the present paper we decided to use the Mt. Wilson system exclusively, on account of the large and homogeneous material contained in the new Mt. W. catalogue[6].

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[6] W. S. Adams, A. H. Joy, M. L. Humason, and A. M. Brayton, *Ap. J.*, **81,** 187, 1935; *Mt. W. Contr.*, No. **511.**

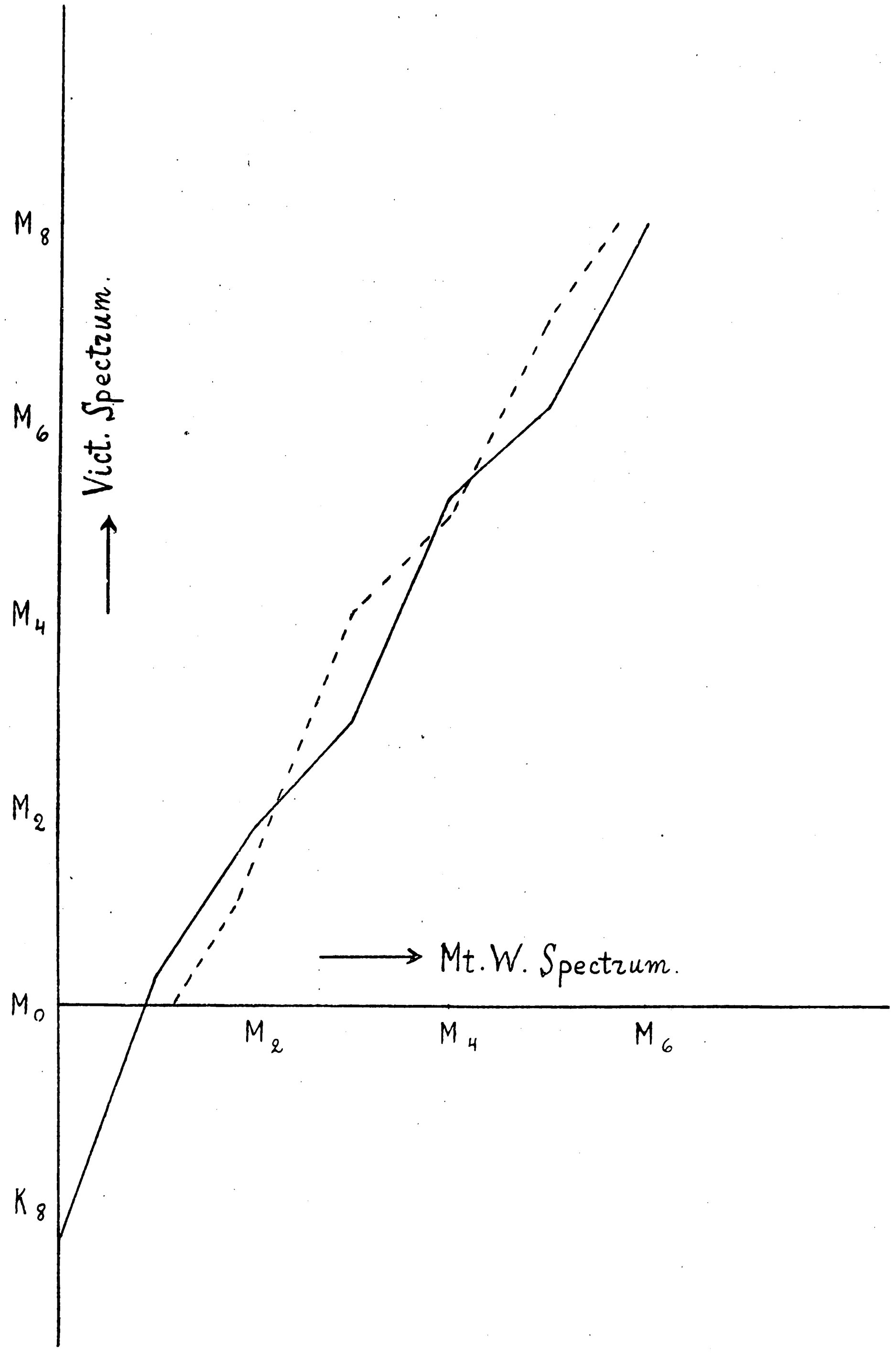

Fig. 2. Correlation of Victoria and Mt. Wilson spectral type for *M* stars.

3. The Colours of Giants and Supergiants.

For the computation of the correlation between colour and spectrum, we used the Catalogue of Colour Indices by Öpik[3]; the system of colour indices considered here is therefore Öpik's system.

The correlation between the "true" colour, and the spectrum is found by adding to the mean observed colour the *TiO* correction ΔC.

It is well known that the colour index of a certain spectral subdivision depends upon the absolute magnitude. We find that the colours of giant *M* stars fall into two groups: the giants, and the supergiants. The colour index of the latter is about 0.46 mag. greater than of ordinary giants.

The supergiants could be distinctly separated only among *M*0 — *M*2 stars (cf. Section 5 below).

Table V contains the computed averages. The fourth column gives the number of stars; the fifth column — the probable error $\pm\frac{\Delta_0}{\sqrt{n}}$ of the average colour index, where $\Delta_0 = \pm 0.10$ is the computed probable deviation of one colour index from the mean of the given spectral subdivision; the sixth column — the true effective temperature computed from formula (2).

Table V.

Dependence of Observed *(C)*, and Corrected *(C')* Colour upon Spectrum.

Sp.	$\overline{C}$	C'	n	$\frac{\Delta_0}{\sqrt{n}}$	T
			Giants		
*M*0	1.63	1.71	23	±0.02	3300
*M*1	1.65	1.79	12	±0.03	3220
*M*2	1.72	2.00	23	±0.02	3000
*M*3	1.70	2.18	23	±0.02	2850
*M*4	1.53	2.32	15	±0.03	2740
*M*5	1.40	2.58	6	±0.04	2550
*M*6	1.30	2.95	4	±0.05	2320
			Supergiants		
*M*0	2.08	2.16	2	±0.07	2860
*M*1	2.10	2.24	2	±0.07	2800
*M*2	2.20	2.48	3	±0.06	2620

Fig. 3 represents the spectrum-colour correlation graphically; the full lines correspond to the corrected colour, the broken ones — to the observed colour; the upper curves refer to the supergiants, the lower ones — to the ordinary giant stars. As shown by the graphs, the change in the spectrum-colour curve produced by the *TiO* correction is enormous.

It may be interesting, after finding the correlation between

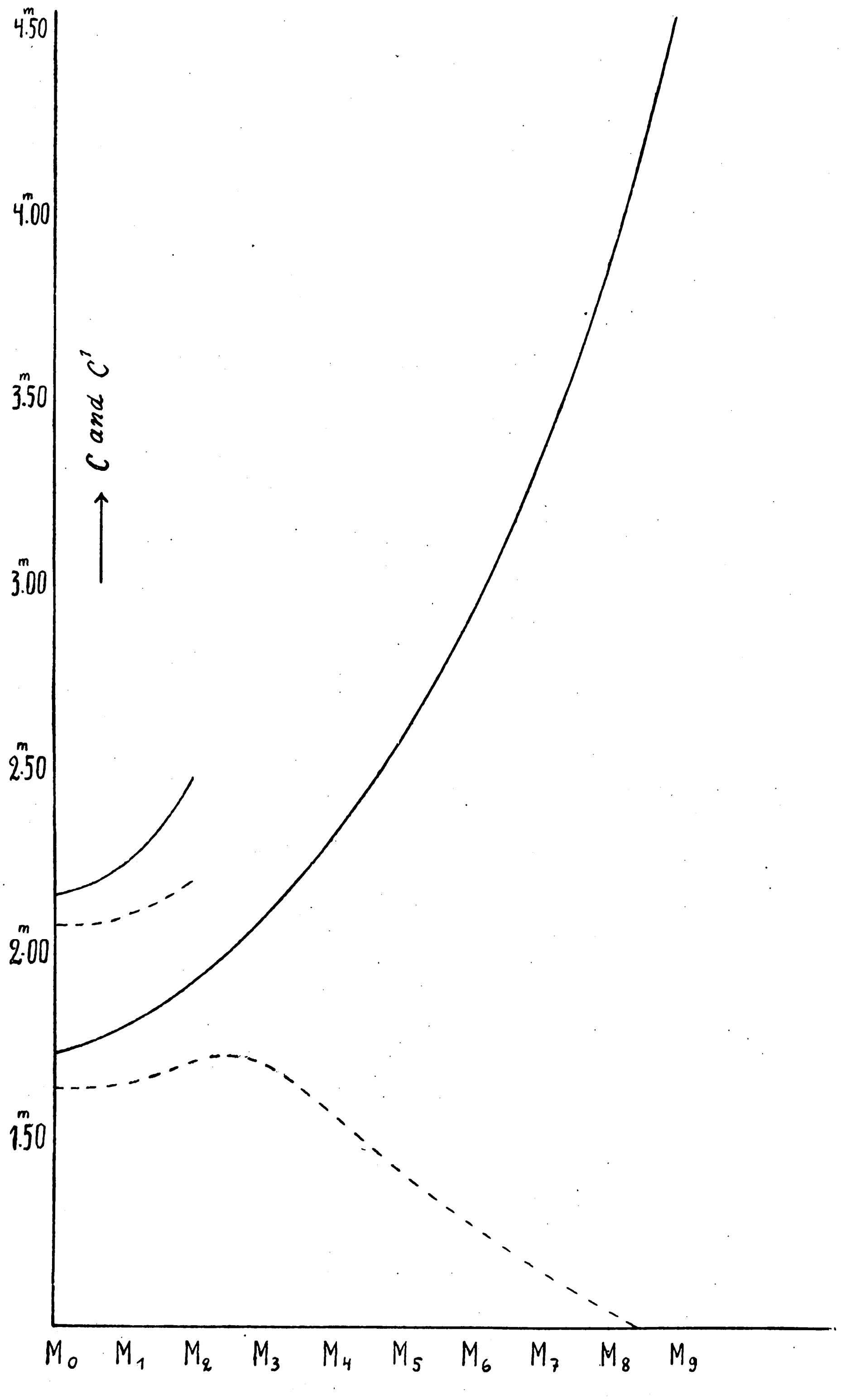

Fig. 3. Colour — spectrum diagrams of *M* giants. Full lines — with *TiO* correction; broken lines — without the correction. The upper pair of curves refers to supergiants, the lower pair — to "ordinary" giants.

corrected colour and spectrum for the *M* stars, to trace a curve representing the same correlation for a greater spectrum interval. This correlation is given by Fig. 4, the upper branch referring to ordinary giant stars, the lower branch, to dwarf stars. A remarkable feature is the small change of colour in the region *GO—KO*, and the much more rapid change in the later spectra.

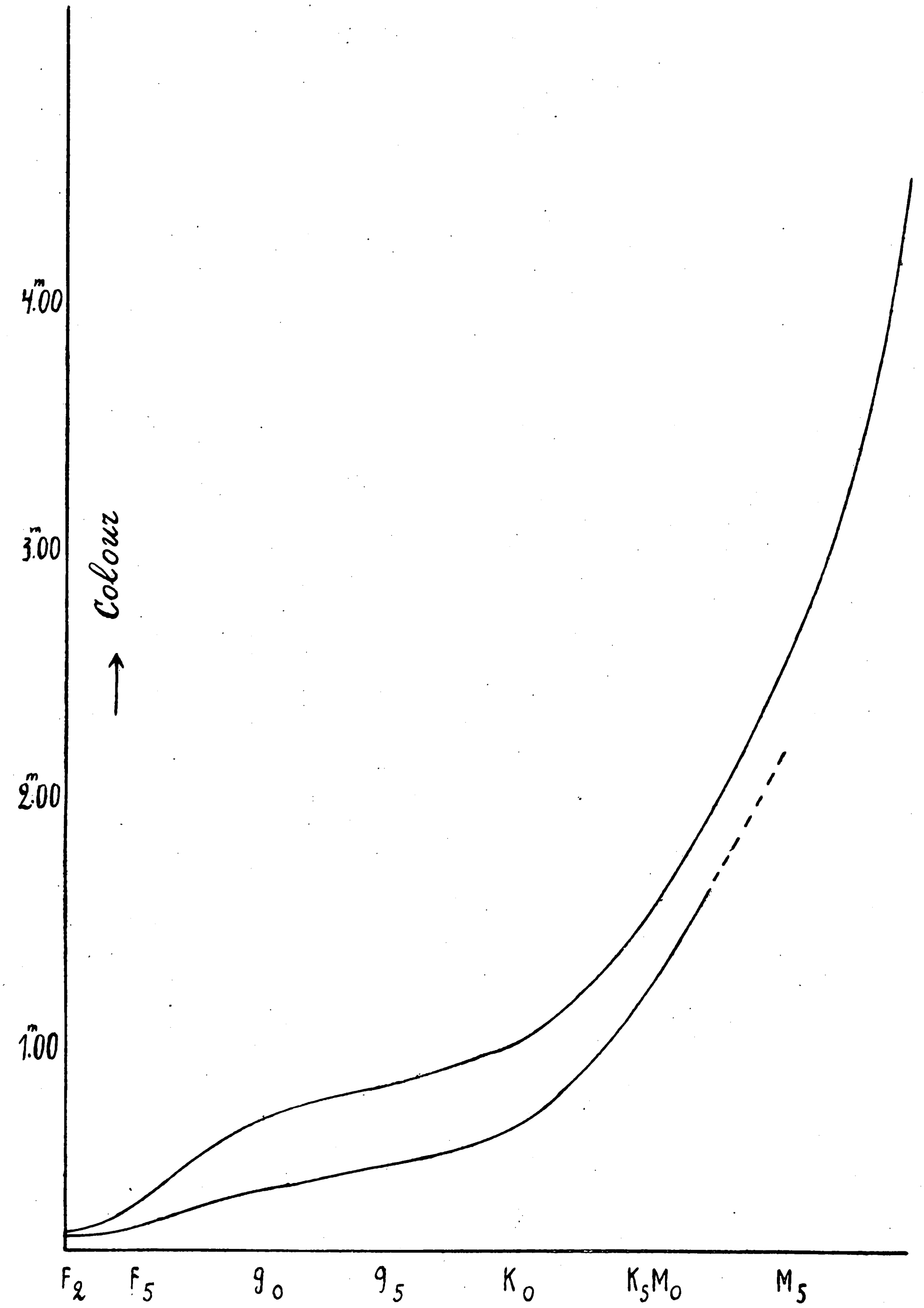

Fig 4. General colour-spectrum diagram, with *TiO* correction for the *M* stars, Upper curve — giants, lower cuive — dwarfs.

The data for colour indices of *F* and *G* stars are taken from[3], Table 37; although the spectral data there are from Mt. W. Contr. **199**, a recomputation with the new Mt. Wilson spectra would hardly make any difference. For the *K* stars, however, it might be advisable to recompute the mean colour indices, taking into account the slight modification in the new Mt. Wilson classification, where the giant series runs *K*5, *M*0, *M*1, etc., while for the dwarfs is given an intermediate group *K*6. Actually we excluded group *K*6 from the present graph.

4. The Colour of Dwarfs.

Whereas for the *M* giants we have a comparatively great number of individual colours at our disposal, the observational data for *M* dwarfs are rather scarce. Table VI contains the individual colour indices of the few *M* dwarfs, present in Öpik's catalogue.

Table VI.

Individual Colour Indices of *M* Dwarfs.

H. D. Sp.	88230 *M*0	107596 *M*0	201092 *M*0	(Cin 1885) *M*0	95735 *M*2	119850 *M*2
C	1.36	1.23	1.52	1.26	1.45	1.32

On the basis of such incomplete data we can hardly decide on the nature of the spectrum-colour curve for dwarfs in the region *M*0—*M*5. However, it seems to be possible to obtain a sufficiently reliable guess for the mean colour indices of *M* dwarfs in another manner.

As shown by Fig. 4, the difference between the colours of giants and dwarfs increases until *G*0 is reached; from here on, the difference is about constant (at least the variation is very small, from 0.30 to 0.36 mag.). This indicates a nearly constant difference of the reciprocal temperature in the region *G*0—*M*2, the temperature of giants being lower.

The best thing we can do is to suppose that this constant difference in colour between ordinary giants and dwarfs persists also in the region *M*2—*M*5. For the constant difference in the colour index we assume the value 0.33, over the whole interval *G*0 — *M*5.

As to the *TiO* correction, although the data for ΔC are derived mostly on the basis of giant stars, we assume for *M* dwarfs the same *TiO* effect; it seems to be the best assumption possible, taking into account that the strength of the *TiO* bands is the only criterion of the spectral classification of *M* stars, both of giants and of dwarfs.

Thus, we may obtain at once the corrected average colour of *M* dwarfs by subtracting the constant value 0.33 from the average corrected colour of giants. Table VII shows these values.

Table VII.

Assumed Mean Corrected Colour Indices of *M* Dwarfs.

Sp.	*M*0	*M*1	*M*2	*M*3	*M*4	*M*5
C'	1.38	1.46	1.67	1.85	1.99	2.25

The broken line in Fig. 4 represents this guessed portion of the spectrum-colour correlation.

5. The Visual Absolute Magnitude of Giants and Supergiants.

Table VIII contains average absolute magnitudes $\overline{M}_v$ of the giant *M* stars of the Mt. Wilson catalogue[6] after applying Van Rhijn's corrections[7]; $M'_v = \overline{M}_v - \Delta C$ is the average absolute magnitude corrected for *TiO* absorption.

The fourth column contains the number of stars; the fifth, the probable error of the mean absolute magnitude. This probable error cannot be based on the supposed individual accuracy of the spectroscopic magnitudes; each spectroscopic magnitude contains a zero point accidental error, depending upon the number of proper motion stars upon which the calibration curve of the spectroscopic absolute magnitudes is based. Thus, in estimating the probable error of the mean, we must take as the basis the individual accuracy of an absolute magnitude determined from

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[7] P. J. van Rhijn, *M. N.*, **92**, 744, 1932.

Table VIII.

Mean Observed ($\overline{M}_V$) and Corrected (M'_V) Visual Absolute Magnitudes of *M* Giants.

Sp.	$\overline{M}_V$	M'_V	n	p. e.
		Giants		
$M\,0$	-0.26	-0.34	75	± 0.16
$M\,1$	-0.20	-0.34	52	± 0.19
$M\,2$	-0.38	-0.66	63	± 0.18
$M\,3$	-0.62	-1.10	57	± 0.19
$M\,4$	-0.64	-1.43	43	± 0.21
$M\,5$	-0.57	-1.75	21	± 0.30
$M\,6$	-0.62	-2.27	12	± 0.40
$M\,7$	-0.7	-2.9	1	± 1.4
		Supergiants		
$M\,0$	-2.82	-2.90	8	± 0.50
$M\,1$	-2.78	-2.92	4	± 0.70
$M\,2$	-3.11	-3.39	8	± 0.50

proper motion alone. Actually we assumed for the p. e. the value $\pm \frac{1.4}{\sqrt{n}}$ magnitudes.

The supergiants can be distinctly separated from ordinary giants only in the spectrum interval $M\,0 - M\,2$; on account of the *TiO* correction, ordinary giants of later spectra get supergiant corrected absolute magnitudes; also, in the Mt. Wilson list, there are no *M* giants later than $M\,2$ whose luminosity considerably exceeds the general mean for the same spectral subdivision.

6. The Bolometric Magnitudes and Masses.

For the actual range of temperatures considered here we replace the member $10 \log T$ in formula (1) successfully by a linear expression; then we get from (1) and (2) the bolometric absolute magnitude (M_B) as function of the visual absolute magnitude and the colour as follows:

$$M_B = M'_V - 1.25\,C' + 1.09 \quad . \; . \; . \; . \; . \quad (4).$$

For the mean masses of giant stars we used Eddington's mass-luminosity relation. Table IX contains the values found for the bolometric absolute magnitudes as computed from (4) and the mass logarithms of giant stars.

Table IX.

Mean Bolometric Absolute Magnitudes (M_B) and Hypothetical Mass Logarithms (log μ) of *M* Giants.

Sp.	M_B	log μ
	Giants	
M 0	−1.39	0.86
M 1	−1.49	0.89
M 2	−2.07	1.01
M 3	−2.73	1.15
M 4	−3.24	1.27
M 5	−3.89	1.42
M 6	−4.87	1.68
M 7	−5.93	1.95
	Supergiants	
M 0	−4.51	1.55
M 1	−4.63	1.58
M 2	−5.40	1.78

As to the *M* dwarfs, Öpik and Gabovitš have pointed out[1] that the empirical logmass — bolometric magnitude relation seems to be nearly linear. Using the photographic determinations of masses given by Huffer[8] and the trigonometric parallaxes from Schlesinger's[9] new catalogue, we find this linear relation to be

$$\log \mu = 0.647 - 0.133\, M_B \quad . \quad . \quad . \quad . \quad . \quad (5).$$

From (4) and (5) we computed the typical bolometric absolute magnitudes and masses of *M* dwarfs, using the mean corrected visual absolute magnitudes and the data for colour indices given by Table VII. Table X gives the computed values.

Table X.

Typical Bolometric Absolute Magnitudes, and Hypothetical Masses of *M* Dwarfs.

Sp.	*M* 0	*M* 1	*M* 2	*M* 3	*M* 4	*M* 5
M_B	7.62	7.90	8.10	8.26	8.36	9.02
μ	0.43	0.40	0.37	0.35	0.34	0.28

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[8] R. Huffer, *Ap. J.*, **80**, 269, 1934.

[9] F. Schlesinger, L. F. Jenkins, General Catalogue of Stellar Parallaxes, Yale Univ. Obs., 1935.

As shown by the data, the mass decreases slightly with the advancing M spectrum; it may be remarked that the TiO correction makes the decrease of the mass considerably smaller than it would be without this correction.

7. The Radii and Densities of Giants and Supergiants.

For the radius (R) of a star we have the following equations (in units of the sun):

$$\log R = 0.5 \log J + 0.5 \log \frac{E_\odot}{E} \quad . \; . \; . \; . \; . \; . \quad (6),$$

$$\log J = 0.4 \, (M_\odot - M_v) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7),$$

$$\text{and } \log \frac{E_\odot}{E} = \frac{C_2}{\lambda} \left(\frac{1}{T} - \frac{1}{T_\odot} \right) \log e \quad . \; . \; . \; . \; . \quad (8),$$

(neglecting the member — 1 in Planck's formula (8), which is permissible in the case of M stars). $M_\odot = 4.85$; $T_\odot = 5810^0$ (assumed); $C_2 = 14300$; $\lambda = 0.56$; the equations (2), (6), (7), and (8) give us for the radius

$$\log R = 0.570 \; C' - 0.2 \; M'_v + 0.722 \quad . \; . \; . \; . \quad (9).$$

For the density (ϱ) we have the simple relation

$$\log \varrho = \log \mu - 3 \log R \quad . \; . \; . \; . \; . \; . \quad (10),$$

Table XI.

Mean Radii and Densities of M Giants.

Sp.	log R	R ($\odot = 1$)	ϱ ($\odot = 1$)	p. e.
		Giants		
$M0$	1.765	58.2	$3.7 \cdot 10^{-5}$	±0.048
$M1$	1.810	64.6	$2.9 \cdot 10^{-5}$	±0.057
$M2$	1.994	98.6	$1.1 \cdot 10^{-5}$	±0.051
$M3$	2.184	153	$4.0 \cdot 10^{-6}$	±0.053
$M4$	2.329	213	$1.9 \cdot 10^{-6}$	±0.061
$M5$	2.542	348	$6.0 \cdot 10^{-7}$	±0.078
$M6$	2.858	721	$1.3 \cdot 10^{-7}$	±0.103
$M7$	3.182	1520	$2.5 \cdot 10^{-8}$	—
		Supergiants		
$M0$	2.532	340	$9.0 \cdot 10^{-7}$	±0.121
$M1$	2.583	383	$6.8 \cdot 10^{-7}$	±0.166
$M2$	2.812	649	$2.2 \cdot 10^{-7}$	±0.118

where log μ is given by Table IX, and log R determined by (9). In Table XI are given the mean values of radii and densities, computed from (9) and (10).

The probable error of the mean logarithm of radius, as given in the fifth column, is computed from

$$\text{p. e.} = \pm \sqrt{\frac{(0.57\,\Delta_{0c})^2}{n_c} + \frac{(0.2\,\Delta_{0M})^2}{n_M}},$$

where Δ_{0c} and Δ_{0M} (values given above) denote the probable deviations of individual colour and absolute magnitude respectively, n_c and n_M — the numbers of colours and absolute magnitudes; for the probable deviation of an individual log R we have

$$\Delta_0 = \pm \sqrt{(0.57\,\Delta_{0c})^2 + (0.2\,\Delta_{0M})^2} = \pm 0.286.$$

Table XII.

Stars Having the Greatest Radii and the Smallest Densities.

Star	α_{1900}	δ_{1900}	m	Sp.	C	ΔC	C'	M'_V	M_B	Mass	Radius		Density
											Sun=1	A.U.	
	h m		m		m	m	m	m	m				
182917	19 21.9	+50° 3′	7.1	*M* 7	1.1	2.2	3.3	−2.9	−5.93	88.7	1520	7.07	$2.51 \cdot 10^{-8}$
α Her	17 10.1	+14 30	3.6	*M* 5	*1.57*	*1.26*	2.83	−3.86	−6.31	104.7	1280	5.94	5.00
μ Cep	21 40.4	+58 19	4.4	*M* 2	*2.42*	0.28	2.70	−4.08	−6.37	106.4	1200	5.58	6.21
α Ori	5 49.8	+ 7 23	0.9	*M* 2	*1.98*	*0.27*	2.25	−5.07	−6.79	128.2	1040	4.84	$1.13 \cdot 10^{-7}$
119 Tau	5 26.3	+18 31	4.7	*M* 2	*2.21*	0.28	2.49	−4.28	−6.30	99.8	995	4.62	1.01
178770	19 4.8	+39 0	7.6	*M* 6	1.30	1.65	2.95	−2.95	−5.55	69.0	984	4.57	$7.24 \cdot 10^{-8}$
α Sco	16 23.3	−26 13	1.5	*M* 1	*2.11*	*0.15*	2.26	−4.75	−6.48	107.4	914	4.25	$1.41 \cdot 10^{-7}$
33664	5 6.7	−11 58	5.9	*M* 6	1.30	1.65	2.95	−2.75	−5.35	61.8	897	4.17	$8.55 \cdot 10^{-8}$
139216	15 31.8	+15 26	6.8	*M* 6	*1.21*	1.65	2.86	−2.95	−5.44	64.3	875	4.06	9.59
1760	0 16.7	−20 37	5.6	*M* 6e	1.30	1.65	2.95	−2.55	−5.15	55.3	818	3.80	$1.01 \cdot 10^{-7}$
101153	11 33.3	+ 8 41	5.5	*M* 6	*1.66*	1.65	3.31	−1.45	−4.50	40.6	792	3.68	$8.17 \cdot 10^{-8}$
30 Her	16 25.4	+42 6	5.0	*M* 6	*1.24*	1.65	2.89	−2.55	−5.07	52.6	759	3.53	$1.21 \cdot 10^{-7}$
14488	2 15.3	+56 39	8.7	*M* 6	1.30	1.65	2.95	−2.25	−4.85	47.0	713	3.41	1.30
163990	17 53.9	+45 23	6.2	*M* 6	1.30	1.65	2.95	−2.05	−4.65	42.2	650	3.02	1.54
88517	10 7.4	− 9 49	8.5	*M* 6	1.30	1.65	2.95	−2.05	−4.65	42.2	650	3.02	1.54
56 Leo	10 50.8	+ 6 43	6.0	*M* 5	*1.22*	*1.43*	2.65	−2.73	−4.96	48.2	601	2.80	2.22
7861	1 13.2	+55 48	8.9	*M* 6	1.30	1.65	2.95	−1.65	−4.25	34.3	541	2.52	2.17
14404	2 14.5	+57 24	8.6	*M* 2	2.20	0.28	2.48	−2.98	−4.99	48.0	538	2.50	3.08
202380	21 10.2	+59 41	7.1	*M* 2	2.20	0.28	2.48	−2.78	−4.79	43.1	491	2.28	3.64
49331	6 42.8	− 8 53	5.3	*M* 0	2.08	0.08	2.16	−3.68	−5.29	54.0	488	2.27	4.66
R Lyr	18 52.3	+43 49	4.3	*M* 5	*1.22*	*1.28*	2.50	−2.58	−4.61	39.2	460	2.15	4.02
168574	18 15.4	−24 58	6.4	*M* 5	1.40	1.18	2.58	−2.28	−4.41	35.6	445	2.07	4.05
45 Ari	2 50.2	+17 56	5.9	*M* 6	*1.07*	*1.59*	2.66	−1.99	−4.22	32.6	434	2.02	4.00
35601	5 20.8	+29 50	8.0	*M* 0	2.08	0.08	2.16	−3.38	−4.99	45.8	425	1.98	5.98
δ^2 Lyr	18 51.0	+36 46	4.5	*M* 4	*1.53*	0.79	2.32	−2.89	−4.70	40.1	419	1.95	5.46
44537	6 17.2	+49 20	5.4	*M* 0	*2.14*	0.08	2.22	−2.88	−4.57	36.9	366	1.70	7.50
213310	22 25.4	+47 12	4.6	*M* 0	*2.01*	0.08	2.09	−3.18	−4.70	38.8	355	1.65	8.69
60414	7 29.2	−14 18	5.4	*M* 3e	1.70	0.48	2.18	−2.78	−4.41	33.7	331	1.54	9.29

In addition, it may be of some interest to compute the individual data for stars having the greatest radii and smallest densities. Table XII contains the results of this computation. The columns of this table give: (1) the name or H. D. number of the star; (2) and (3) right ascension and declination for 1900; (4) the H. D. apparent visual magnitude; (5) the Mt. Wilson spectrum; (6) the apparent colour index; observed values[3] are in italics; (7) the *TiO* correction; observed individual data[1] for ΔC are in italics; (8) the corrected colour index; (9) the corrected absolute visual magnitude; (10) the bolometric absolute magnitude; (11) the hypothetical mass; (12) the radius in units of the sun; (13) the radius in astronomical units; (14) the density (sun = 1).

As shown by the data in Tables XI and XII, radii greater than 300 times the sun's radius appear to be quite numerous in our lists; a great number of long period variable stars, not considered here, must doubtlessly possess radii and densities of the same order of magnitude.

8. Comparison with Other Results.

It might be interesting to compare our theoretical data with the results obtained from two different methods: stellar radiation measurements by Pettit and Nicholson[4], and interferometer measurements of stellar diameters by Michelson and Pease[10]. There are only three stars with measured interferometer diameters available for comparison.

In Table XIII, column five, are computed the angular diameters of these three stars, from the equation

$$\log D = 0.57\, C' - 0.2\, m'_v - 2.31 \quad . \; . \; . \; . \; . \; . \quad (11),$$

deduced from (9), by assuming $D_\odot = 1922''.4$; m'_v is the apparent visual magnitude, corrected for *TiO* absorption.

Pettit and Nicholson computed diameters by two independent methods — the "water-cell absorption" (the fraction of radiation absorbed by the water-cell), and the "heat-index" (visual minus radiometric magnitude). As follows from the considerations in this paper, the "heat-index" must be also

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[10] A. Michelson and F. Pease, *Mt. W. Contr.*, No. **203,** 1922; also[4], Table VI.

Table XIII.

Comparison with Interferometer Data.

Star	Sp.	m'_V	C'	Diameter					
					Michelson and Pease		Pettit and Nicholson		
				Computed from equation (11)	Assuming an uniformly illuminated disk	Corrected for limb darkening	From water-cell absorption	From apparent heat index	From corrected heat index
		m	m						
α Scorpii	*M* 1	1.38	*2.26*	0″.050	(0″.040)	0″.049	0″.062	(0″.065)	0″.061
α Orionis	*M* 2	0.65	*2.25*	0 .069	(0 .047)	0 .057	0 .071	(0 .076)	0 .069
α Herculis	*M* 5	2.22	*2.83*	0 .070	(0 .030)	0 .038	0 .065	(0 .090)	0 .060

corrected for *TiO* absorption; contrary to what happens with the "ordinary" colour-index, the apparent "heat-index" yields evidently a temperature systematically too low; numerically, the "heat-index" must be corrected by subtracting ΔC (cf. Table II) from the figures published by Pettit and Nicholson.

As shown by columns 5, 8, and 10 of Table XIII, the agreement between the "water-cell", the "corrected heat-index", and our computed diameters is excellent.

It may be added that the "water-cell absorption", depending upon a wide portion of the spectrum (λ 0.4 — 1.4 μ), must be influenced by *TiO* absorption in much less a degree than the narrow visual portion of the spectrum. Further, we notice that for α Herculis the diameter computed from the "heat-index" is led to agreement with the two other methods only after applying the *TiO* correction (cf. Table XIII, columns 9, and 10).

As to the diameters, measured with the interferometer, they were derived on the assumption of an uniformly illuminated disk. However, the stars considered possess undoubtedly a limb darkening; an application of Wien's law in connection with the theory of radiative equilibrium shows that the darkening of these stars in the visual region λ 0.56 μ must be the same as for the wave-lengths $\lambda\lambda$ 0.27, 0.27, and 0.23 respectively in the sun. From Lindblad's theoretical considerations[11], confirmed by observations for longer wave-lengths, it follows that the limb darkening for these wave-lengths, at a distance 0.95

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[11] B. Lindblad, *Uppsala Univ. Årsskrift* 1920, Mat. **1**, page 22.

from the centre (the intensity of the centre = 1), is 0.226, and 0.178; these values correspond to $n = 0.64$, and 0.74 in the darkening formula adopted by Michelson and Pease[10]. Hence, according to tables given by these authors, the published interferometer results must be increased by 21, and 25 per cent respectively. The measured diameters, corrected for limb darkening, are given in Table XIII, column seven. As shown by the data in columns 5, and 7, the agreement for the two first stars is quite satisfactory. As to α Herculis, the discrepancy is sensible; a reduction of the assumed *TiO* correction by 20 per cent (or taking 80 per cent of the assumed correction) would have given perfect agreement*; such a single fact, however, cannot be regarded as a sufficient reason for introducing a systematical correction.

The difference for α Herculis may be entirely due to the uncertainty in colour index of this star, influenced by its *F* type companion; taking instead of the individual colour of α Herculis the mean corrected colour index for spectrum *M*5, 2.58 mag., the result would have yielded a better agreement, the ratio $D_{\text{comp.}} : D_{\text{interf.}}$ being then 1.26 only, instead of 1.82 in the first case.

9. The *M* Giant Stars as Black Body Radiators.

In the above mentioned paper on Stellar Radiation[4] Pettit and Nicholson pointed out the considerable deviations from black body radiation for *M* stars, ascribing it to *TiO* absorption. As will be shown below, their view seems to be correct; in fact, *TiO* absorption accounts almost entirely for all the observed deviations.

In Figure 5 the corrected "heat-indices" (cf. preceding section) of the *M* giant stars in Pettit and Nicholson's list are plotted against the observed "water-cell absorption". The increasing "water-cell absorption" corresponds to a decreasing temperature, i. e., to an advancing *M* spectrum. The full line represents the theoretical black body curve (for λ 0.555; cf.[4], Table IV). As to the long period variables at minimum, their

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* Without *TiO* correction the diameter becomes $0''.008$, or one-fifth of the observed value; this shows how important the introduction of the *TiO* correction is, especially for the later *M* stars.

exact spectral class is mostly unknown. In order to obtain the necessary *TiO* correction for these stars, we assumed, on the basis of known cases, their spectra to be about three spectral subdivisions later than the spectra at maximum; on Figure 5 these stars are represented by open circles.

Figure 5 shows an unexpectedly satisfactory agreement between the empirical "corrected heat-index" — "water-cell absorption" curve, and the theoretical one. The large deviation from the theoretical curve, found by Pettit and Nicholson for *M* stars, practically disappears. Remarkable is, however, the systematical negative deviation of the "heat index" for the hotter *M* type stars, and the opposite effect for the lower temperatures.

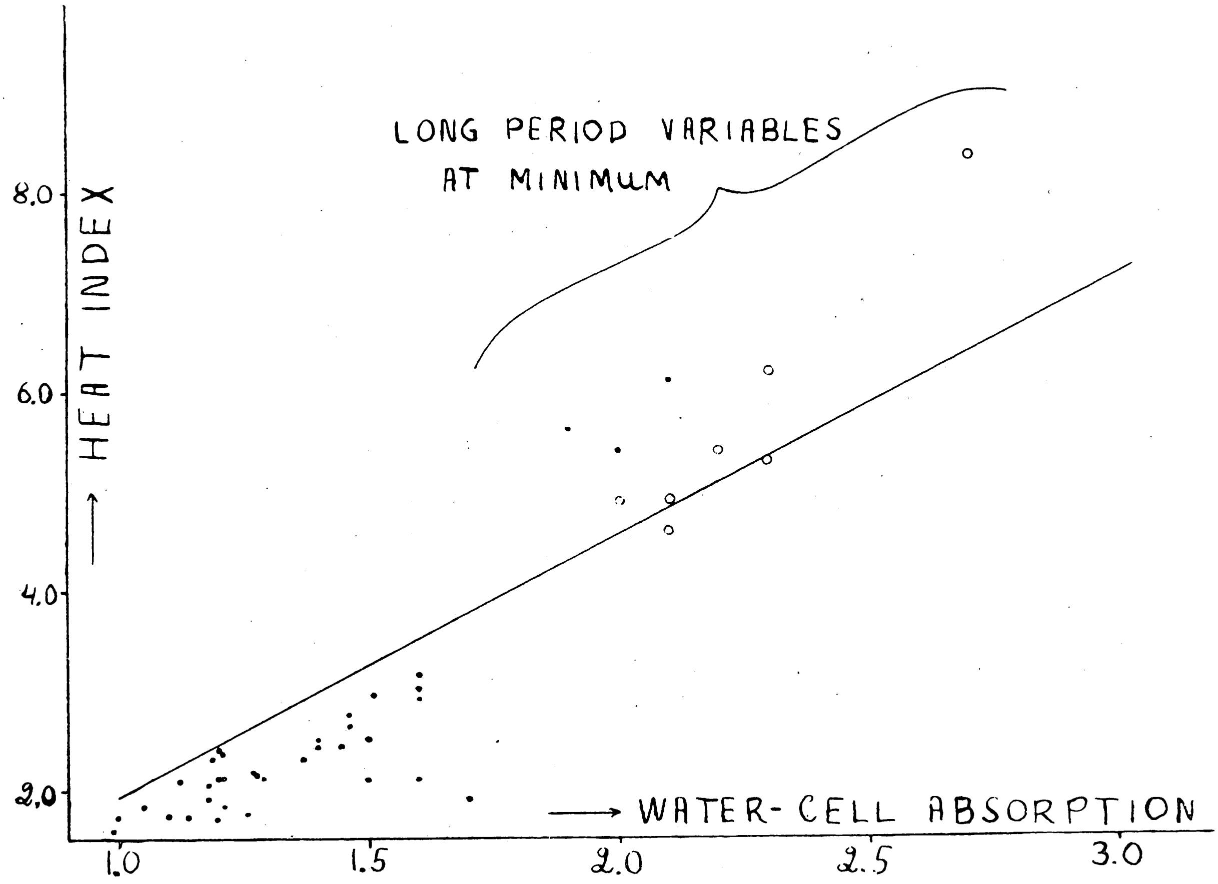

Fig. 5. Correlation of "corrected heat index" and "water-cell absorption" for *M* stars. The full line represents the theoretical black body curve (λ 0.555).

10. The Densities of Dwarf Stars.

From equations (4), (5), (9), and (10) we get for the density of an *M* dwarf:

$$\log \varrho = 0.467\, m_p + 2.335 \log \pi - 2.010\, C' + 0.671 \quad . \; . \; . \; . \; . \quad (12).$$

Table XIV*.

Individual Densities of *M* Dwarfs.

Star	α_{1900}	δ_{1900}	Sp.	m_p	$\pi \pm \Delta\pi$	log ϱ	ϱ ($\odot = 1$)
	h m						
ADS 48 *B*	0 0.4	+45° 16′	*M* 0	10.6	0″.089±0″.005	0.39	2.46
20 *C* 16	11.8	+40 23	*M* 0	*10.1*	.083 9	0.08	1.22
ADS 246 *A*	12.7	+43 27	*M* 3	9.5	.284 5	0.12	1.30
" " *B*	12.7	+43 27	*M* 5	11.6	.284 5	0.29	1.95
ADS 433—440 *A*	26.3	+66 42	*M* 3	10.9	.103 7	—0.26	0.55
Boss 168 *B*	43.0	+57 17	*M* 0	8.7	.182 5	0.23	1.68
C 142	1 0.4	+63 24	*M* 1	*10.2*	.068 5	-0.23	0.59
C 238	36.8	+63 20	*M* 0	9.5	.071 7	—0.35	0.45
C 251	48.0	—22 56	*M* 0	10.2	.081 8	0.11	1.29
Boss 617 *B*	2 37.4	+48 48	*M* 3	11.2	.078 5	—0.41	0.39
ADS 2218 *B*	49.7	+26 28	*M* 1	*10.9*	.059 9	—0.04	0.91
C 404	3 1.2	+ 1 36	*M* 0	10.2	.058 6	-0.23	0.59
C 499	38.2	+68 21	*M* 0	10.5	.054 9	—0.16	0.69
C 554	4 2.2	—21 6	*M* 0	10.4	.064 11	-0.04	0.92
Boss 984 *C*	10.8	— 7 49	*M* 5e	12.1	.202 3	0.17	1.49
C 594	29.8	+52 42	*M* 1	*10.0*	.092 7	—0.02	0.96
Comp. Boss 1246	5 10.0	+45 44	*M* 2	11.9	.075 4	0.24	1.74
C 705	26.4	— 3 42	*M* 3	10.2	.168 4	-0.10	0.80
C 771	6 6.4	—21 49	*M* 2	10.1	.183 6	0.30	2.02
20 *C* 393	31.5	+17 38	*M* 1	*10.9*	.100 5	0.50	3.13
C 837	49.5	+40 13	*M* 0	*9.8*	.031 4	—1.05	0.089
20 *C* 418	7 13.0	+33 2	*M* 2	*11.5*	.048 9	—0.40	0.40
Boss 1979 *C*	28.2	+32 5	*M* 1e	10.4	.073 3	—0.06	0.86
20 *C* 475	8 27.4	+67 38	*M* 0	*10.6*	.084 7	0.33	2.15
20 *C* 490	43.0	+36 53	*M* 1	*11.9*	.028 9	—0.34	0.46
ADS 7067 *A*	46.0	+71 11	*M* 1	10.7	.093 4	0.32	2.10
" " *B*	46.0	+71 11	*M* 1	10.9	.093 4	0.42	2.63
Boss 2404 *B*	52.4	+48 26	*M* 1	12.1	.067 12	0.64	4.40
Boss 2469 *(A)*	9 7.6	+53 7	*M* 0	9.4	.162 3	0.44	2.77
Boss 2470 *(B)*	7.6	+53 7	*M* 0	9.4	.162 3	0.44	2.77
20 *C* 532	25.8	+36 46	*M* 2	*11.6*	.065 7	—0.04	0.91
C 1167	46.2	—11 49	*M* 2	*11.3*	.076 5	0.02	1.04
20 *C* 553	48.8	+63 16	*M* 1	*10.5*	.065 7	-0.14	0.73
C 1218	10 5.3	+49 58	*M* 0	8.2	.220 8	—0.18	0.67
C 1225	7.5	+53 1	*M* 0	*10.6*	.045 9	—0.30	0.50
C 1244	14.2	+20 22	*M* 4e	*10.9*	.193 8	0.10	1.24
C 1246	15.7	— 0 58	*M* 0	10.2	.043 11	—0.53	0.29
20 *C* 582	25.5	+46 3	*M* 1	*10.2*	.048 13	—0.58	0.26
Boss 2935	57.9	+36 38	*M* 2	9.0	.388 6	0.01	1.02
C 1349	11 0.5	+44 2	*M* 2	*10.1*	.174 8	0.26	1.81
ADS 8083 *A*	5.6	+31 0	*M* 1	10.2	.081 7	—0.05	0,89
" " *B*	5.6	+31 0	*M* 2	11.8	.081 7	0.27	1.87
C 1364	6.4	—14 26	*M* 0	10.6	.058 11	—0.04	0.91
C 1383	14.8	+66 23	*M* 1	*10.7*	.120 5	0.58	3.80
20 *C* 695	12 14.4	+28 56	*M* 2	*11.6*	.051 9	—0.28	0.52
C 1551	16.9	+42 42	*M* 0	10.4	.070 13	0.19	1.56
20 *C* 713	26.3	+ 9 22	*M* 1	*11.1*	.050 10	—0.12	0.76
C 1633	45.6	— 0 13	*M* 0	10.0	.099 8	0.22	1.66
C 1661	55.2	— 2 10	*M* 0	10.8	.063 10	0.14	1.36

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* For the meaning of the abbreviations in column 1 cf.[6].

Table XIV. Continued.

Star	α 1900		δ 1900	Sp.	m_p	$\pi \pm \Delta\pi$		log ϱ	ϱ (⊙ = 1)
	h	m							
ADS 8861 *A*	13	14.9	+35°40′	*M* 1	*10.9*	0″.086	+0″.009	0.34	2.19
AD 8887 *A*		18.9	+29 45	*M* 0	10.8	.067	11	0.20	1.58
C 1784		40.2	+18 20	*M* 1	10.4	.084	10	0.08	1.19
C 1786		40.7	+15 26	*M* 2	9.9	.191	8	0.40	2.49
ADS 9090 *B*		58.5	+46 49	*M* 4	10.8	.079	6	—0.86	0.14
C 1885	14	17.6	+30 6	*M* 0	*9.9*	.073	10	—0.06	0.88
βGC 6869 *A*		21.1	+24 6	*M* 1	*10.8*	.056	7	—0.14	0.72
" " *B*		21.1	+24 6	*M* 2	*11.1*	.056	7	—0.43	0.38
C 1905		25.6	— 8 12	*M* 1	10.7	.072	7	0.06	1.15
Boss 3812 *(B)*		51.6	—20 58	*M* 2	9.5	.172	4	—0.03	0.92
C 1989		55.3	—10 43	*M* 0	11.3	.057	8	0.27	1.85
C 2012	15	3.1	+25 18	*M* 0	*11.2*	.077	7	0.52	3.35
20 *C* 920		8.8	— 3 26	*M* 0	*10.7*	.030	6	—0.66	2.17
20 *C* 923		14.2	— 7 21	*M* 5	*12.1*	.152	6	—0.11	0.77
20 *C* 968	16	2.9	+34 55	*M* 0	*11.4*	.046	8	0.10	1.25
20 *C* 995		24.7	—12 24	*M* 5	*11.2*	.255	5	—0.01	0.98
C 2238		41.4	+33 41	*M* 0	9.9	.113	6	0.31	2.04
C 2251 *A*		50 1	— 8 9	*M* 3e	*11.1**	.151	4	0.22	1.65
20 *C* 1014		54.1	+25 55	*M* 2	*11.0*	.086	9	—0.04	0.91
Boss 4342		59.8	— 4 54	*M* 0	9.2	.083	4	—0.33	0.47
C 2278	17	0.0	— 4 56	*M* 3	*11.2*	.100	5	—0.15	0.70
20 *C* 1023 *A*		9.2	+45 52	*M* 4	10.8	.144	7	—0.25	0.56
C 2297		9.9	+42 28	*M* 1	10.8	.032	6	—0.71	0.20
C 2347		33.4	+18 37	*M* 1	*11.2*	.113	9	0.76	5.69
C 2354		37.0	+68 26	*M* 3	*10.7*	.213	5	0.38	2.39
20 *C* 1062		40.9	+43 26	*M* 3	11.7	.102	7	0.10	1.26
Boss 4497 *B*		42.5	+27 47	*M* 3	*11.2*	.109	6	—0.07	0.86
Barnard's st.		53.0	+ 4 25	*M* 5	10.8	.545	3	0.57	3.75
20 *C* 1095	18	32.4	+45 39	*M* 2	10.9	.086	9	—0.08	0.82
ADS 11632 *A*		41.8	+59 27	*M* 4	10.5	.282	4	0.28	1.93
" " *B*		41.8	+59 27	*M* 5	10.8	.282	4	—0.09	0.81
C 2463		44.5	+17 20	*M* 1	*10.4*	.059	4	—0.28	0.52
C 2475		53.1	+ 5 48	*M* 1	*10.7*	.079	5	0.16	1.44
C 2556	19	29.6	+ 4 21	*M* 1	*11.0*	.081	5	0.32	2.10
C 2648	20	13.8	+76 55	*M* 0	10.6	.075	7	0.22	1.65
C 2707		51.3	+61 48	*M* 2	10.0	.138	5	—0.03	0.94
20 *C* 1250 *A*		56.2	+39 41	*M* 3e	11.3	.097	8	—0.14	0.73
Boss 5434 *(B)*	21	2.4	+38 15	*M* 0	7.8	.299	3	—0.03	0.92
C 2757		11.4	—39 15	*M* 1	8.0	.257	7	0.10	1.24
C 2790		24.5	—12 56	*M* 0	10.7	.048	7	—0.19	0.65
ADS 15972 *A*	22	24.4	+57 12	*M* 3	10.7	.258	4	0.58	3.79
20 *C* 1370 *A*		28.7	+53 16	*M* 1	12.1	.036	7	0.01	1.03
20 *C* 1382		42.5	+43 49	*M* 5e	11.3	.207	7	—0.17	0 67
C 3001		55.0	—23 4	*M* 1	9.0	.125	7	—0.17	0.67
C 3014		59.4	—36 26	*M* 2	8.8	.278	6	0.12	1.32
C 3124	23	44.0	+1 52	*M* 2	*10.5*	.167	6	0.40	2.51
C 3143		53.5	+46 10	*M* 0	*11.2*	.058	6	0.24	1.73
C 3161		59.5	—37 51	*M* 3	9.7	.222	7	—0.04	0.90

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* The value for m_p given by Willis[10] is 10.4; the star was found double by Kuiper (1934) (cf.[6], page 290); therefore we give the corrected magnitude of one component.

Using the Mt. W. spectra, and the data for trigonometric parallaxes compiled in the new catalogue by Schlesinger[9], we computed individual densities for ninety-seven *M* dwarfs. Table VII furnishes us with the values of C' in (12) (in a few cases individual observed colours as given in Table VI could be used); the photographic apparent magnitude m_p is taken partly from recent determinations by Willis[12], partly computed from the Harvard visual magnitude, or (in the case of double stars) from the statistical investigation on double star magnitudes by Öpik[13]. Table XIV contains the results. The columns give: (1) the designation of the star; (2) and (3) the position for 1900; (4) the Mt. W. spectrum; (5) the photographic apparent magnitude; italics indicate the magnitudes determined by Willis[12]; (6) the trigonometric parallax from Schlesinger's catalogue[9] and its probable error; (7) the logarithm of the density; (8) the density (sun = 1).

On the basis of Table XIV we computed the logarithmic mean densities of *M* dwarfs for each spectral subdivision as given in Table XV. The fourth column contains the number of stars, the fifth, the probable error of the mean density logarithm, computed from $\pm \frac{\Delta_0}{\sqrt{n}}$, where Δ_0 denotes the probable individual deviation of the density logarithm which we found to be $\pm$ 0.224.

Thus the cosmic spread (Δ) in the density logarithm for a given spectral group of *M* dwarfs is

$$\Delta = \pm \sqrt{0.224^2 - (2.335\,\Delta_{\log \pi})^2 - (0.467\,\Delta_m)^2 - (2.01\,\Delta_c)^2},$$

where $\Delta_{\log \pi}$ is the p. e. of $\log \pi$, Δ_m — the photometric error in the apparent magnitude, and Δ_c — the probable deviation of the individual colour from the assumed mean (Δ_c is a cosmic error). We assume: $\Delta_{\log \pi} = \pm 0.063$ (computed); $\Delta_m = \pm 0.2$; $\Delta_c = \pm 0.06$. This gives for the cosmic spread in $\log \varrho$, $\Delta = \pm 0.071$, or ± 17 per cent. Within the uncertainty of the adopted component errors, which are likely to be underestimated, a still smaller spread of $\log \varrho$ appears to be possible.

Thus, on the basis of the data of Table XV, and from the value for Δ found above, we arrive at the unexpected and highly

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[12] H. C. Willis, *Mt. W. Contr.*, No. **502,** 1934.

[13] E. Öpik, *Publ. Tartu Obs.*, **25.** 6, 1924.

Table XV.
Mean Densities of *M* Dwarfs.

Sp.	$\overline{\log \varrho}$	ϱ	n	$\frac{\Delta_0}{\sqrt{n}}$
M 0	+0.02	1.05	31	±0.04
M 1	+0.05	1.12	26	±0.05
M 2	+0.04	1.10	17	±0.05
M 3	+0.02	1.05	12	±0.06
M 4	−0.18	0.66	4	±0.11
M 5	+0.09	1.23	7	±0.08

significant conclusion with respect to the densities of *M* dwarfs: the mean densities of class *M* dwarf stars are nearly equal to the density of the sun over the whole spectral range from *M*0 to *M*5, and the cosmic spread of their densities is very small.

This result gives us a new proof of the reliability of the adopted scale of *TiO* correction, because a false colour index system would hardly lead to such a small cosmic spread in the densities of *M* dwarfs.

As to the frequency curve of density logarithms, it shows an excellent agreement with a Gaussian error curve, the agreement being actually better than expected on the basis of the law of chance. This circumstance appears to be quite natural, from our standpoint expressed above, according to which the frequency curve obtained represents more likely the error distribution, than the true distribution of densities.

In conclusion, I wish to express my sincere thanks to Dr. Ernst Öpik for steady helpful advice, and friendly criticism.

Tartu, December 1935.