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Determination of Colour-Equivalents of Stars According to Tikhoff's Method

with Some Considerations Concerning the Relation of Colour
to Absolute Magnitude and Spectrum.

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1. Introduction.

The present paper is the result of a preliminary investigation regarding the applicability of Tikhoff's method¹⁾ to the determination of colour-indices. The method consists in a utilization of the chromatic aberration of the objective for purposes of separating radiation of different wave-lengths. In front of the objective is placed a circular diaphragm which screens the central portion of the objective, leaving free a ring-shaped zone. An extrafocal photograph, obtained with such a diaphragm, will consist of the following two principal elements: a ring, corresponding to the radiation for which the objective is achromatized, and a central condensation, or a more or less sharply defined point, corresponding to the radiation which has its focus in the plane of the plate. The photographic intensity of the ring and the point may be determined separately, and the difference will then give the colour-equivalent. A very important particular of the method is the circumstance that the point represents the combined effect of two different kinds of radiation, one of a shorter, the other of a longer wave-length than the wave-length of achromatization. A substantial feature of the present investigation consists in taking account of this effect of blending, without using a colour-filter.

Some considerations regarding the relation of colour to absolute magnitude and spectrum are added; the material consisting of only 104 stars is small, and the conclusions arrived at are of a preliminary character; an investigation relating to the same subject is to be carried on here on a much larger scale.

1) Извѣстія Русск. Общ. Люб. Міровѣдѣнія, т. V № 3 (1916), p. 119; Astr. Nachr. B. 218, p. 145 (1923). Also by Tamm, Astr. Nachr. B. 216 p. 331 (1922). An investigation on the same subject, carried on at the same time as our investigation, appeared lately: Veröff. Berlin-Babelsberg, B. V H. 2, by B. Sternberk.

Nevertheless, even our small material is sufficient to show the principal features of the relation of colour to absolute magnitude, confirming thus the results of F. H. Seares¹⁾; this is due chiefly to the fact that the said relation is of an individual character; few stars are sufficient to show the phenomenon, if the colour-equivalents are determined accurately enough.

2. Method of Observation and Reduction.

The photographs were obtained chiefly in spring, 1923, with the 160 mm Petzval camera, focal length 792 mm.

For photometric purposes the dispersion of the "longitudinal spectrograph," i. e. the chromatic aberration ought to be the smallest possible, as otherwise there will arise unnecessary loss of light; from the standpoint of smallness of the chromatic aberration the Petzval system of objectives may be regarded as very satisfactory; the objective being achromatized for the photographically most efficient wave-length, the focus of yellow radiation differs by 1.4 mm from the photographic focus. It may be remarked that the size of the objective is of little importance in Tikhoff's method, as with the increasing aperture and length of focus the linear measure of the chromatic aberration increases also, and the light will thus be distributed over a larger surface; therefore, little gain in the limiting magnitude may be expected from the increase of linear aperture of the objective. On the other hand, very promising appears the method if used with a large reflector, where an additional lens, placed near the focus, artificially introduces chromatic aberration; the amount of the latter may in this case be varied to desire.

The photographs were taken partly on Hauff Ultra Rapid (*UR*), partly on Hauff Flaviv (*Fl*) plates, all of the same emulsion. The *Fl* are orthochromatic plates, and the long-wave radiation which produces the point on these plates at 1.40 mm out of focus is estimated to correspond approximately to effective $\lambda = 630 \mu\mu$. The *UR* are ordinary plates, with an effective wave-length of photographic radiation perhaps slightly greater than $430 \mu\mu$, and practically non-sensitive for "visual" radiation; the

1) Mt. Wilson Obs., Comm. to Nation. Acad. № 59 (1919). The phenomenon was first noted by W. S. Adams and by Van Rhijn.

point on the *UR* plates proved to be due exclusively to short-wave radiation, with an effective wave-length about $365\mu\mu$ at 1.50 mm out of focus. The plates were developed during 6 minutes in Methol-Hydrochinon, at a practically constant temperature.

The rotating plate-holder allowed of obtaining many photographs on the same plate. From 2 to 16 different stars or fields were photographed on the same plate. Of the bright stars ordinarily 4 exposures of varying time (e. g. from 4^s to 20^s) were obtained on the same plate, whereas of the faintest stars which require long exposures only one photograph was made. The atmospheric absorption was determined on each day separately from several low stars photographed specially for this purpose.

In all 34 plates with over 1000 exposures, ranging from $0^s.5$ to 1 hour, were obtained. As the intention was to study the influence of the distance from focus upon the intensity of the images, the photographs were taken at different distances from the focus; an additional correction for distance was thus introduced, which, however, did not affect perceptibly the accuracy of the colour-equivalents derived; only the extreme values of distance could not be used for determining the colour-equivalents.

The photometric estimates were made with the aid of scales superposed on the plate, film against film. The scales were similar to the images compared with them, and the point of the star was compared with the point of the scale, the ring of the star — with the ring of the scale; such a method of comparison practically eliminates the photographic Purkinje effect. Each estimate was repeated with the scale turned by 180° . When only one image was obtained on a plate, the number of estimates was doubled. The total number of estimates exceeds 4000.

All *UR* images were compared with a single scale, formed by a series of exposures of τ Bootis, the exposures increasing in the ratio 1:2.

The *Fl* images showed a considerable variety of shape, according to spectral type and distance from focus, and therefore it appeared difficult to compare them with a single scale. In fact the following 9 scales were used, each scale consisting of a series of exposures increasing in the ratio 1:2.

$$\begin{array}{l} \text{Scales } \alpha, \\ d = 0.363 \end{array} \left\{ \begin{array}{l} 1. \quad \delta \text{ Andromedae, Sp. } K_2 \text{ (H.D.)} \\ 2. \quad \omega \text{ Andromedae, Sp. } F_5 \text{ (")} \\ 3. \quad \sigma \text{ Arietis, Sp. } B_5 \text{ (")} \end{array} \right.$$

$$\begin{array}{l} \text{Scales } \beta, \\ d = 0.300 \end{array} \left\{ \begin{array}{l} 1. \\ 2. \quad \text{(Same stars as above)} \\ 3. \end{array} \right.$$

$$\begin{array}{l} \text{Scales } \gamma, \\ d = 0.226 \end{array} \left\{ \begin{array}{l} 1. \\ 2. \quad \text{(Same stars as above)} \\ 3. \end{array} \right.$$

Here d denotes the diameter of the ring as determined on a Repsold measuring machine; the distance from focus equals $5d$.

Each star was compared with the scale which had images most similar to the image of the star. About one-half of the Fl images were compared with two scales. Such stars common to two scales, as well as a direct comparison of the scales inter se furnished a means for determining the systematic differences of the scales.

As the images of a scale, especially the points, may be subject to accidental variation, produced by changes of atmospheric transparency or by a non-uniformity of the plate, the scales required some kind of calibration. The calibration was executed with the aid of a great number of photographs with varying time of exposure. In fact, all fields, where 3 or 4 exposures of the same star were obtained, were used for calibration. The rings and the points of each scale were calibrated separately, and to the different Fl scales systematic corrections were added to reduce them to a uniform scale. In this way each scale reading could be converted into an effective logarithm of exposure of an ideal scale. It may be remarked that this proceeding takes account of real irregularities of the scales as well as of the subjective decimal equation of the observer.

The rings of all Fl scales, and the points of 8 Fl scales could thus be reduced to a uniform scale of rings, or of points respectively. As to scale α , 1 (δ Andromedae), the central image had not the shape of a point, but of a small ring, the secondary ring, and could not therefore be put aside with the other scales. Generally, when the diameter of the ring exceeded 0.340 mm, late type stars (from G_5 and later) showed the

secondary ring, with a faint point at the centre. The secondary rings, practically due to purely long-wave radiation, were treated separately; in fact, only the colour-equivalent of one star (*Bo* 7493) was determined independently from the secondary ring, whereas other 6 cases were used to determine the effect of distance from the focus.

The estimates were thus expressed in one of the following scales of ideal exposure-logarithms:

- 1) Scale *UR* of rings;
- 2) Scale *UR* of points;
- 3) Scale *Fl* of rings;
- 4) Scale *Fl* of points;
- 5) Scale *Fl* of secondary rings (for late-type stars and $d > 0.340$). The diameters of the rings were determined on a Repsold measuring machine.

So far no assumption regarding the laws of photographic photometry were made. As to these laws, the choice must be made between Kron's or Halm's exact formulae, and their particular case — Schwarzschild's law. A discussion of the photographic magnitudes derived from the rings indicated that no perceptible deviation from constancy of Schwarzschild's exponent p exists for the entire range of magnitudes covered by the observations, i. e. from $m = 2$ to $m = 9$. Schwarzschild's formula was therefore adopted. From stars with known photographic magnitudes¹⁾ the most probable values of p for photographic radiation (ring) were found as follows:

$$Fl, p = 0.808 \pm 0.004 \text{ (p. e.)}$$

$$UR, p = 0.816 \pm 0.006 \text{ (p. e.)}$$

In using photographic magnitudes of different sources, they were reduced conventionally to King's scale with the aid of corrections given by Hertzsprung²⁾; to King's magnitudes a constant correction of -0.03 was added which represents the residual correction of the particular group of stars here considered.

1) Determined by Hertzsprung (Bulletin Astr. Inst. Neth. №35), King (Harv. Ann. 76 №6) and Schwarzschild (Göttingen Aktinometrie). Only directly determined photographic magnitudes were used, except in the case of some stars of spectrum A_0 (109 Virginis, λ Ophiuchi).

2) Loc. cit.

By multiplying the effective exposure-logarithms by $-\frac{0.4}{p}$ the scale readings were converted into stellar magnitudes. It was assumed that p is the same for the three kinds of radiation here considered; this assumption is justified by the general character of the present observational data, and is confirmed by some investigations made elsewhere¹⁾, though others have found a variation of p with wave-length. The same value of p was therefore used for the point and the ring²⁾. The difference of magnitude between the point and the ring gives the colour-equivalent, which for the *UR* plates may be used directly as the colour-equivalent of $365\mu\mu - 430\mu\mu$; the *Fl* colour-equivalents need a correction for superposed short-wave radiation.

Besides the colour-equivalents, photographic magnitudes could also be determined; for this purpose a plate constant, c , was added to the preliminary magnitudes; this constant was also derived from the stars with known photographic magnitude mentioned above. The colour-equation of the *Fl* rings proved to be the same as the colour-equation of King's magnitudes, whereas the *UR* rings showed a slightly greater effective wave-length; therefore, on *UR* only stars of spectrum G_0 or earlier were used for the determination of the constant c , and the following correction for colour-equation was added to the *UR* photographic magnitudes:

$$\text{corr.} = +0.07 (m_{\text{phtgr.}} - m_{\text{vis.}}) - 0.03.$$

The *Fl* colour-equivalents, except those derived from the secondary ring, were corrected for short-wave radiation on the assumption of an additive effect of the two kinds of radiation; of course, the existence of the Purkinje phenomenon indicates that this assumption is not generally acceptable; but in our particular case of a blend of radiation at $365\mu\mu$ and $630\mu\mu$ the assumption proved valid; no perceptible difference of gradation was revealed by the point of the *B* and the *K* stars, the for-

1) H. Rosenberg. *Photogr. Untersuchung der Intensitätsverteilung in Sternspektren* (Halle 1914), p. 77.

2) In determining the colour-equivalents the round value of $p = 0.80$ was used throughout, whereas the photographic magnitudes were derived with the exact values of p given above.

mer containing about $\frac{2}{3}$, the latter less than 10% of short-wave radiation¹).

Let x and y be the colour-equivalents corresponding to the difference of intensity in the spectral regions violet — blue (photographic) and blue — yellow respectively; let x be determined from an *UR* plate, and let the uncorrected (blended) colour-equivalent of the *Fl* plate be F . Assuming the photographic intensity of blue radiation equal to 1, the photographic intensity in the other two spectral regions on a *Fl* plate will equal a^{-x+b} for the violet, and a^{+y} for the yellow, where $a=2.512$. The constant shift between the *UR* and the *Fl* colour-equivalents is here denoted by b .

On the assumption of the additive properties of the blend we have:

$$a^{+F} = a^{+y} + a^{-x+b} \quad . \quad . \quad . \quad (1),$$

whence y may be determined, if F , x and b are given.

The constant b was derived from the mean data of observation for two groups of stars, for which both F and x were determined, and for which the difference in y was assumed equal to the difference of colour-index as determined from other sources. These groups were: 1) 4 stars of spectrum B_0-B_5 (ρ Leonis, τ Herculis, 90 Leonis, η Ursae Majoris); 2) 9 stars of spectrum A_0-A_3 (38 Lyncis, δ , Θ Leonis, γ , δ Ursae Maj., ξ Virginis, ξ Bootis, γ Coronae, β Serpentis).

The average data for these stars were:

	x	Mean F	colour-index (Harv. Scale)
B_0-B_5	$+0.44$	-0.06	-0.14
A_0-A_3	-0.30	-0.40	$+0.08$

Let y_1 and y_2 be the *Fl* corrected colour-equivalents of the two groups; owing to the smallness of the difference in colour-index, we assume the same difference for y ; thus,

$$y_2 = y_1 + 0.22 \quad . \quad . \quad . \quad (a).$$

The data for the two groups give two equations of the form (1), which together with (a) furnish three equations for the

1) As the measure of radiation the photographic effect produced on *Fl* plates is here assumed.

determination of the three unknowns, y_1 , y_2 and b .

The groups of stars chosen are especially fit for the determination of b , as, with a small range in the visual colour-index, the *UR* colour-equivalent shows a considerable range, produced by the intense hydrogen absorption in the violet spectral region of the *A* stars.

In this way the value of $b = -0.89$ was found; the round value, $b = -0.90$, was finally adopted and used in the computation of y , according to formula (1), for stars of spectrum A_0 or later¹⁾. For earlier spectra the short-wave radiation prevails, and the correction becomes uncertain; therefore, for stars of spectrum B_0 to B_9 , as well as for α Coronae and 12 Canum Venaticorum, the problem was inverted, and the *UR* colour-equivalent was also derived from the *Fl* plates, assuming the colour-index as given by the difference of photographic and visual magnitude. The *UR* colour-equivalents derived from the *Fl* plates required a residual correction of $+0.17$.

The different determinations of atmospheric absorption are summarized in table 1.

The data of this table indicate that, according to the transparency, the days were not of equal quality. The results of one day, April 8, are not included in the table; on this day haze was present all during the observations; the apparent brightness of stars seemed diminished by 0.5—1.0 magnitudes, as estimated by the eye, and varied considerably; some photographs were nevertheless obtained on this day, with the purpose to determine the influence of haze upon the colour-equivalents. The interesting conclusion was arrived at that, although the apparent photographic magnitude as indicated by the intensity of the ring varied within a range from 0.0 to 1.8 magnitudes below the normal value, the colour-equivalents showed no systematic deviation from the values determined on other days. Thus haze on this day exerted absorption equally upon all wave-lengths. It was therefore assumed that the differential absorption remained the same during the whole series of photographs. On the contrary, the absolute absorption for blue radiation showed perceptible variation, evidently related to the remarks concerning the quality

1) For some stars no *UR* photographs were obtained; in such cases the mean *UR* colour-equivalent of the corresponding spectrum was assumed.

Table 1. *Atmospherical Absorption.*

For comparison only stars of approximately the same spectrum were used.

a) *Absorption of Ring* (about $\lambda = 430 \mu\mu$).

Date, 1923	March 26	March 31	April 6	April 10	April 12	April 13	April 22	April 23
Absorption, st. mg. per unit of air-mass	0.28	0.33	0.32	0.61	0.31	0.39	0.33	0.32
Stars used (all stars were pho- togra- phed during meridian passage)	$C 30$ Hydrae; β Librae; μ Serpentis; γ Crateris; δ Corvi	$C 30$ Hydrae	γ Corvi; β Librae	γ Crateris; α_2 Librae	γ Crateris; γ Corvi; α_2 Librae	γ Crateris; γ Corvi; β Librae	γ Corvi; α_2 Librae	δ Corvi; α_2 Librae; μ Serpentis
	ι , γ , δ Ursae Maj.; γ , ζ Bootis; γ Herculis; α , β Coronae; δ , θ Leonis; 38 Lyncis	38 Lyncis	δ , 93 Leonis; γ Ursae Min.; γ Coronae	42 Leo Min.; β Virginis; θ Bootis; η Ursae Maj.	θ , 90 Leonis; γ Ursae Maj.; β Coronae	θ , 93 Leonis; ϵ Ursae Maj.; α , θ Coronae	δ Ursae Maj.; ζ Bootis; 109 Virgin.	90 Leonis; 12 Can. Ven.; ζ Bootis; θ Coronae; τ Herculis
Remarks on the condition of the sky made during obser- vation	Moon.	Full Moon. Very clear sky.	— — —	Sky trans- parent; but $1\frac{1}{2}$ hour after finishing clouds	Of excellent quality	Transpa- rency prob- ably satisfac- tory, though about 14^h S.T. and later haze towards south (hazy wall of $60'$ height)	— — —	— — —

Table 1. Continued.

b) Differential Absorption of *UR* Colour-Equivalent, *Point* — *Ring* (violet — blue). As no differences produced by the haziness of the sky could be detected the data are grouped according to the absorption star:

Absorption Star	Comparison Stars ¹⁾	Differential Absorption Point-Ring st. mg.	Difference in <i>Sec z</i>	Number of Nights	Coefficient of Differential Absorption
γ Crateris	β Virgin.; 42 Leo Min.; β Serpentis	+0.74	2.79	2	+0.26
γ Corvi	θ , δ Leonis; γ Ursae Maj.; γ , β Coronae	+0.36	2.79	2	+0.13
α_2 Librae	θ , δ Leonis; δ , ζ_1 Ursae Maj.; β , γ Coronae; γ , ζ Bootis 109 Virgin.; χ Serpentis	+0.67	2.37	3	+0.28
μ Serpentis	42 Leo Min.; 50 Bootis	+0.14	1.00	2	+0.14
Sum		+1.91	8.95	9	— — —
Mean		— — —	— — —	— — —	+0.213

c) Differential Absorption of *Fl* colour-equivalent, *Ring* — *Point*²⁾ (blue — yellow). Stars of Sp. B_8 or earlier not used. The data were corrected for difference in colour-index of absorption star and comparison stars.

	Diameter of Ring		Mean
	0.270	0.330	
Mean Coeff. of Differ. Absorption	+0.12	+0.15	+0.13
Nights	3	3	— — —

of the day; it appears that on April 10 and 13 some haze was present, whereas the remaining nights were perfectly clear³⁾.

1) Nearly of the same spectrum and distance from focus as the absorption star.

2) Freed from short-wave radiation.

3) During the last hour on March 26 (second half of Neg. 107) haze appeared also, which, however, was not noted by the observer; the photographic brightness appeared to be suddenly decreased by 0.5 — 0.8 st. mg., and for photographic magnitudes these observations could not be used.

Table 2. Corrections for Diameter (*d*).
1) *Intensity of the Ring: Fl, or UR.*

1000 <i>d</i> , mm	180	190	200	210	220	230	240	250	260	270	280	290	300
Corr., st. mg.	+0.76	+0.64	+0.54	+0.45	+0.38	+0.31	+0.26	+0.20	+0.15	+0.10	+0.07	+0.04	0.00

1000 <i>d</i> , mm	310	320	330	340	350	360	370	380	390	400	420	440	460
Corr., st. mg.	-0.06	-0.12	-0.20	-0.27	-0.32	-0.37	-0.42	-0.48	-0.53	-0.58	-0.69	-0.79	-0.89

2) *UR* Colour-Equivalent, *violet* — *blue*. The correction was excellently represented by the formula:
corr. = — *k* (*d* — 0.300) st. mg., where *k* depends on the colour of the star:

Colour-Index, Harv. Scale ($m_{\text{photogr.}} - m_{\text{vis}}$)	+1.8	+1.7	+1.6	+1.5	+1.4	+1.3	+1.2	+1.1 ... 0.2	+0.1 ... 0.0	Sp. $B_0 - B_9$ c. i. < -0.10
	8.	7.	6.	5.3	4.5	4.0	3.7	3.5	3.0	
k										0.0

3) *Fl* Colour-Equivalent, *blue* — *yellow*. The corrections proved to be exactly the same for all spectra, which points at practically monochromatic radiation. For *d* > 0.340 the table relates only to spectra earlier than *G*₅.

1000 <i>d</i> , mm	180	190	200	210	220	230	240	250	260	270
Correction + constant ¹⁾ , st. mg.	+1.06	+1.05	+1.05	+1.04	+1.03	+1.02	+1.01	+1.00	+0.99	+0.99

1000 <i>d</i> , mm	280	290	300	310	320	330	340	350	360	370
Correction + constant ¹⁾ , st. mg.	+1.01	+1.04	+1.12	+1.23	+1.36	+1.51	+1.68	+1.87	+2.05	+2.20

1) The constant was added to make the colour-equivalent of *A*₀ zero.

Table 3. Plates used for photographic magnitudes.

a) UR Plates.

Plate	113. 114	119. 120	122	123	127. 128	130. 131	133	134	138 ¹⁾	139 ¹⁾	140 ¹⁾
Date, 1923	6. IV	10. IV	12. IV	12. IV	13. IV	22. IV	23. IV	23. IV	8. V	8. V	8. V
c, st. mg.	5.54	5.36	5.79	5.63	5.43	5.63	5.55	5.42	5.42	5.59	5.39
Stars Used in the De- termina- tion of c	δ ; 93 Leonis; ξ ; η Ursae Maj.; η Bootis	42 Leo Min.; β Virgin.; δ ; ϵ Ursae Maj.; β Can. Ven.	θ Ursae Maj.; 38 Lynceis; γ Serpentinis; γ ; ζ Her- culis	γ Ursae Maj.; 90; θ Leonis; β Serpentinis; α ; β ; γ Co- ronae	α ; θ Co- ronae	70; 64 Vir- gin.; τ ; θ ; ζ Bootis; χ Herculis; χ Serpentinis	ζ_1 Ursae Maj.; 14 Can. Ven.; ζ Bootis	50 Bootis; θ Coro- nae; τ Hercu- lis; λ Ophi- uchi	ν ; η ; ρ Boo- tis	ρ ; δ Bootis 109 Vir- gin.	δ Boo- tis β Cor.; β Serp.
Stars not Used in the Deter- mination of c ²⁾	ν Ursae Maj.; Boss 3078; Bo 8468; Ci 1553	22 H Camel.; ν ; ϵ Virgin.; 12 Can. Ven. (A); Be A 4874	40 Lynceis; α ; 39; ζ Leon.; 11; 20 Leo Min.; Bo 7493; Ci 1819; Grb. 1616; β Herculis	Boss 3294; Lund 5297; 38 Leo Min.; α Serpentinis; δ ; β Bootis	Be A 5003; δ Bootis; Lal. 29307; 12 Ophi- uchi	Boss 3398; B.D. + 23 ⁰ 2537; ν ; α ; ρ ; γ ; ϵ Bootis; Boss 3867; B.D. + 19 ⁰ 2937; Nic. 3891; B.D. — 0 ⁰ 2943	g Ursae Maj.; α ; ρ ; γ Bootis	β Bootis; η Coro- nae	γ Boo- tis	—	

1) For these three plates late-type stars were also used in determining c. 2) Absorption stars not included.

Table 3. Continued.

b) *F7* Plates.

Plate Date, 1923	135. 136. 29. IV 4. V	106. 109. 115. 121. 126. 129. 26. III 6. IV 10. IV 13. IV 29. IV	107. 108. 110 132. 26. III 31. III 23. IV	111. 125. 31. III 13. IV	124. 10. IV
c, st. mg.	4.53	4.63	4.73	4.83	4.92
Stars Used in the Deter- mination of c	Neg. ρ; ϵ; v; η; ζ; β Bootis; ϵ; 109 Vir- gin.; β Coronae; β Serpentis	Neg. (106) β; δ; i (A) Cancri; o; i Ursae Maj.; ζ Hydrae; 38; 40 Lyncis; η; α; ρ; ζ Bootis. (109) γ; δ Ursae Maj.; ϵ; δ; ζ Vir- gin.; v Bootis. (115) γ Ursae Min.; γ Coronae; γ Ser- pentis; τ; γ; η Herculis. (121) η Ursae Maj.; η, θ Bootis. (126) 93 Leonis; β Can. Ven.; ϵ Ursae Maj.; β Bootis; 64 Virgin. (129) δ Ursae Maj.; Boss 3294; δ; γ Virgin; 14 Can. Ven.	Neg. (107) o; ϵ; μ; η; α; η; ζ; ρ Leonis; μ Ursae Maj. (108) β; ψ; ξ; v Ursae Maj.; δ; θ; β Leonis; v; β Vir- gin. (110) 38; 40 Lyncis; θ Ursae Maj.; o; ϵ; μ Leonis. (132) 90 Leonis; Boss 3867.	η; α; ζ; θ Leonis; μ Ursae Maj.;	Be A 4874; 43 Comae; 70 Virgin.; α Bootis.
Stars not Used in the Determi- nation of c^2)	12 Can. Ven.; γ Bootis	(106) γ; ϵ Bootis; 109 Virgin. (109) 12 Can. Ven. (115) Be A 5003; Nic. 3891; Lal. 29307; λ; 12 Ophiuchi. (121) Lal. 26294. (126) Boss 3078; Boss 3398; B.D. +230 2537. (129) Lund 5297; Boss 3398; B.D. +230 2537.	(107) Ci 1819; Boss 2750. (108) (110) 11 Leo Minoris. (132) Nic. 3352; 12 Can. Ven. (A); B.D. +190 2937.	20; 38 Leo Min.; 39 Leonis; Ci 1819; Boss 2750; Bo 7493; Grb. 1616; Bo 7627; 22 H Camelop.	Bo 8468; Ci 1553.

2) Absorption stars not included.

In the reduction of photographic magnitudes the following coefficients of absorption were used.

Date, 1923	26, 31 III; 6, 12, 22, 23, 29 IV; 4, 8 V	10 IV	13 IV
Mean Absorption, st. mg. per unit of air-mass (λ about 430 $\mu\mu$)	0.31	0.61	0.39

The low value of the coefficient of absorption of the first group is remarkable; in fact, these days were exceptionally clear.

For the differential absorption the average values given in sections b) and c) of table 1 were used.

The *corrections for diameter of the ring*, i. e. for distance from the focus, are given in table 2.

All plates of the same sort showed practically the same colour-sensitiveness, and no correction was applied to the colour-equivalents derived from single plates¹). On the contrary, as stated above, the photographic magnitudes required a correction c varying from one plate to another. Of course, the range of variation of c was small. In determining c , plates which gave similar values of the constant were joined together. Table 3 contains a summary of the adopted values of c together with some other data.

3. Results.

Table 4 contains a summary of the results. The table is self-explanatory.

The Fl colour-equivalent proved to be closely related to the Harvard colour-index ($m_{photgr.} - m_{vis.}$); our colour-equivalents are systematically greater, which apparently is explained by the greater interval of effective wave-length. A perfect radiator should give a linear relation between both sets of colour-equivalents; in our case the relation could be represented, instead of one, by two straight lines:

$$\left. \begin{aligned} J_t &= 0.667 \, Fl \quad . \quad . \quad . \quad . \quad \text{for } Fl \leq 1.50, \text{ and} \\ J_t &= 1.00 + 0.860 (Fl - 1.50) \quad . \quad \text{for } Fl > 1.50 \end{aligned} \right\} (2)$$

Here J_t denotes the colour-index on the Harvard Scale (King's photogr. magn. — $H. R.$), Fl — the colour-equivalent as given

1) In this respect our method seems to be more convenient and more free from systematic errors than the method of exposure-ratios devised by Fr. H. Seares.

Table 4. *Results of Observations.*

In parentheses is given the number of images used in deriving the magnitude, or the colour of a star on the corresponding plate.

Name	1900		Photographic Magnitude		UR Colour-Equivalent ¹⁾		FV Colour-Equivalent ²⁾	
	α	δ	Results of Single Plates	Mean	Single Plates	Mean	Single Plates	Mean
β Cancri	8 11.1	90° 31'	5.16 (3)	5.16	...	...	...	...
α Ursae Maj.	8 22.0	61 03	4.39 (4)	4.39	...	...	1.48 (4)	1.48
δ Cancri	8 39.0	18 31	5.22 (4)	5.22	...	...	1.63 (4)	1.63
γ Cancri A.	8 40.6	29 08	5.07 (4)	5.07	...	...	1.43 (4)	1.43
ζ Hydrae	8 50.1	6 20	4.41 (4)	4.41	...	...	1.57 (4)	1.57
ϵ Ursae Maj.	8 52.4	48 26	3.34 (4)	3.34	...	...	0.21 (4)	0.21
38 Lynceis	9 12.6	37 14	4.07 (4); 3.85 (4); 3.86 (4)	3.93	0.29 (4)	0.29	-0.05 (4); 0.21 (4)	0.08
40 Lynceis	9 15.0	34 49	5.00 (4); 5.08 (4); 4.95 (3)	5.01	1.24 (3)	1.24	2.28 (4); 2.11 (4)	2.20
θ Ursae Maj.	9 26.2	52 08	3.77 (4); 3.68 (4)	3.72	0.06 (4)	0.06	0.65 (4)	0.65
11 Leo Min.	9 29.7	36 16	6.35 (3); 6.21 (2)	6.28	0.12 (2)	0.12	1.24 (3)	1.24
α Leonis	9 35.8	10 21	4.13 (4); 4.20 (4); 4.09 (4)	4.14	0.27 (4)	0.27	0.92 (4); 0.68 (4)	0.80
ϵ Leonis	9 40.2	24 14	4.00 (4); 4.00 (4)	4.00	...	...	1.26 (4); 0.98 (4)	1.12
μ Leonis	9 47.1	26 29	5.41 (4); 5.39 (4)	5.40	...	...	2.07 (4); 1.67 (4)	1.87
20 Leo Min.	9 55.2	32 25	6.00 (3); 6.14 (2)	6.07	0.05 (2)	0.05	0.77 (3)	0.77
η Leonis	10 01.9	17 15	3.60 (4); 3.58 (4)	3.59	...	...	0.28 (4); 0.46 (4)	0.37
α Leonis	10 03.0	12 27	1.24 (4); 1.08 (2)	1.16	-0.11 (4); -0.43 (2)	-0.27	...	...
Grb. 1616	10 04.0	50 00	7.62 (1); 7.45 (1)	7.54	0.02 (1)	0.02	-0.07 (1)	-0.07
β 7493	10 05.2	49 58	8.28 (1); 8.10 (1)	8.19	0.87 (1)	0.87	[1.26 (1)] ³⁾	[1.26] ³⁾
35 Leonis	10 11.0	24 00	6.66 (1); 6.46 (4); 6.90 (4)	6.67	0.16 (3)	0.16	0.89 (2); 0.46 (2)	0.68
ζ Leonis	10 11.1	23 55	3.92 (4); 3.92 (5); 3.82 (4)	3.89	0.54 (4)	0.54	0.41 (4); 0.65 (3); 0.65 (2)	0.57
39 Leonis	10 11.7	23 36	6.20 (3); 6.58 (5)	6.39	-0.08 (4)	-0.08	0.60 (3)	0.60
Boss 2750	10 16.3	41 44	6.32 (1); 6.19 (3); 6.48 (4)	6.33	0.07 (4)	0.07	0.82 (1); 0.66 (3)	0.74
μ Ursae Maj.	10 16.4	42 00	4.97 (4); 5.09 (4)	5.03	1.72 (3)	1.72	2.27 (4); 2.15 (4)	2.21
β 7627	10 21.9	49 19	7.11 (2)	7.11	...	...	0.68 (2)	0.68
ρ Leonis	10 27.6	9 49	3.76 (4); 3.84 (4)	3.80	-0.59 (4); -0.92 (4); -0.60 (4)	-0.70	...	...
38 Leo Min.	10 33.4	38 26	6.53 (2); 6.39 (2)	6.46	0.22 (2); 0.25 (2)	0.24	1.24 (2)	1.24
42 Leo Min.	10 40.3	31 13	5.21 (4)	5.21	0.19 (3); 0.17 (4)	0.18	...	...
β Ursae Maj.	10 55.8	56 55	2.16 (4)	2.16	...	...	0.17 (4)	0.17
22 H Camelop.	10 57.9	36 38	9.08 (1); 8.85 (1)	8.96	0.64 (1); 0.92 (1)	0.78	2.25 (1)	2.25
ψ Ursae Maj.	11 04.0	45 02	4.46 (4)	4.46	...	...	1.82 (4)	1.82
δ Leonis	11 08.8	21 04	2.77 (4); 2.66 (4)	2.72	0.35 (4)	0.35	0.39 (4)	0.39
θ Leonis	11 09.0	15 59	3.36 (4); 3.37 (3); 3.38 (4)	3.37	0.34 (3); 0.37 (4);	0.36	0.15 (4)	0.15
ξ Ursae Maj. comb.	11 12.8	32 06	4.44 (4); 4.45 (4)	4.44	0.04 (4)	0.04	0.99 (4)	0.99
γ Ursae Maj.	11 13.1	33 38	5.19 (4); 5.03 (4)	5.11	1.10 (4)	1.10	2.08 (4)	2.08

1) Violet — blue. 2) Blue — yellow. 3) Determined from secondary ring. This value is supposed to represent the colour-index in the Harvard Scale.

Table 4. Continued.

N a m e	1900		Photographic Magnitude		UR Colour-Equivalent		F7 Colour-Equivalent	
	α	δ	Results of Single Plates	Mean	Single Plates	Mean	Single Plates	Mean
Lal. 26294 . . .	^h 14 17.7	30° 06'	9.83 (1)	9.83			1.83 (1)	1.83
6 Bootis . . .	14 21.8	52 19	4.71 (4); 4.47 (3)	4.59	0.06 (3)	0.06	0.66 (4)	0.66
2 Bootis . . .	14 27.5	30 49	5.15 (3); 5.02 (3); 5.04 (4); 4.95 (10); 5.15 (8); 5.12 (4)	5.07	0.81 (3); 0.90 (4); 0.94 (4); 0.61 (4)	0.82	1.55 (3); 1.69 (4)	1.62
7 Bootis . . .	14 28.1	38 45	3.54 (4); 3.28 (3); 3.52 (2); 3.20 (11); 3.09 (11)	3.33	0.21 (3); 0.16 (2); 0.45 (4); 0.54 (4); 0.34 (2)	0.34	0.51 (4); 0.22 (3); 0.24 (4)	0.32
5 Bootis . . .	14 36.4	14 09	4.01 (4); 4.00 (4); 3.91 (3); 3.82 (12)	3.94	0.32 (4); 0.37 (3)	0.34	0.03 (4); 0.11 (3); 0.24 (4); 0.09 (4)	0.12
8 Bootis comb. . .	14 40.6	27 30	3.69 (4); 3.60 (4); 3.42 (12)	3.57	0.42 (4)	0.42	1.31 (4); 1.53 (4); 1.47 (4)	1.44
109 Virgin. . .	14 41.2	2 19	3.94 (4); 3.96 (4); 3.76 (11); 3.76 (12)	3.86	0.22 (4); 0.37 (4); 0.30 (4)	0.30	-0.11 (4); 0.24 (2); 0.09 (4); -0.17 (4)	0.01
9 Bootis . . .	14 58.2	40 47	4.57 (4); 4.55 (2); 4.42 (4); 4.56 (11)	4.52	0.44 (4); 0.57 (4)	0.50	1.27 (2); 1.42 (2); 1.65 (4); 1.48 (4)	1.46
Boss 3867 . . .	15 07.5	19 21	7.64 (1); 7.50 (3)	7.57	1.18 (1)	1.18	1.70 (3)	1.70
B. D. + 19° 2937 . . .	15 08.2	19 28	8.98 (1)	8.98	1.11 (1)	1.11		
B. D. - 0° 2943 . . .	15 08.2	-0 46	8.85 (1)	8.85	0.30 (1)	0.30		
Nic. 3891 comb. . .	15 08.8	-0 58	7.22 (2); 7.50 (1)	7.36	0.26 (1)	0.26	1.22 (2)	1.22
8 Bootis . . .	15 11.5	33 41	4.45 (3); 4.36 (4); 4.44 (4); 4.50 (8)	4.44	0.20 (3); 0.57 (4); 0.47 (4); 0.41 (4)	0.41		
50 Bootis . . .	15 17.8	33 17	5.28 (4)	5.28	0.16 (4)	0.16		
7 Coronae comb. . .	15 19.1	30 39	5.60 (3)	5.60	0.01 (3)	0.01		
7 Ursae Min. . .	15 20.9	72 11	3.09 (2)	3.09			0.02 (2)	0.02
9 Coronae . . .	15 23.7	29 27	3.96 (4); 3.91 (10); 4.00 (10)	3.96	0.23 (4); 0.26 (4); 0.17 (4); 0.01 (4)	0.17	0.20 (2); 0.24 (4); 0.42 (4)	0.29
6 Coronae . . .	15 23.9	31 42	3.93 (6); 3.99 (4)	3.96	-0.15 (6); -0.23 (4)	-0.19		
4 Coronae . . .	15 30.5	27 03	2.11 (3); 2.13 (4)	2.12	0.22 (3); 0.50 (4); 0.37 (4); 0.24 (3)	0.33		
2 Serpentis . . .	15 37.1	13 10	5.48 (4)	5.48	0.28 (4)	0.28		
7 Coronae . . .	15 38.5	26 37	3.94 (5); 3.77 (3)	3.86	0.11 (3)	0.11	-0.22 (5)	-0.22
9 Serpentis . . .	15 41.6	15 44	3.69 (3); 3.65 (11); 3.53 (8)	3.62	0.39 (3); 0.32 (4); -0.40 (4)	0.37	-0.06 (4); 0.01 (3); 0.10 (4); -0.08 (4)	-0.01
2 Serpentis . . .	15 44.2	18 27	5.99 (4)	5.99	1.52 (3)	1.52		
4 Serpentis . . .	15 44.4	-3 07					-0.02 (4)	-0.02
5 Serpentis . . .	15 45.8	4 47			0.31 (4)	0.31		
2 Herculis . . .	15 49.2	42 44	5.27 (3)	5.27	-0.02 (3)	-0.02		
7 Serpentis . . .	15 51.8	15 59	4.30 (4); 4.37 (4)	4.34	-0.11 (4); 0.10 (4); 0.01 (4); 0.22 (4)	0.06	0.57 (4); 0.61 (4)	0.59
Lal. 29307 . . .	15 59.9	25 31	8.02 (1); 7.61 (1)	7.82	0.51 (1)	0.51	1.38 (4)	1.38
2 Herculis . . .	16 16.7	46 33	3.58 (4); 3.63 (4)	3.60	-0.30 (4); -0.42 (4)	-0.36		
7 Herculis . . .	16 17.5	19 23	3.99 (4); 4.01 (4)	4.00	0.44 (4); 0.22 (4); 0.51 (4); 0.61 (4)	0.44	0.24 (4); 0.28 (4)	0.26
2 Ophiuchi . . .	16 25.9	2 12	3.97 (4); 3.78 (4)	3.88	0.33 (4)	0.33	0.17 (4)	0.17
9 Herculis . . .	16 25.9	21 42	3.78 (4)	3.78	0.17 (4)	0.17		
12 Ophiuchi . . .	16 31.1	-2 07	6.49 (2); 6.28 (1)	6.38	0.45 (1)	0.45	1.10 (2)	1.10
5 Herculis . . .	16 37.5	31 47	3.53 (3)	3.53	-0.01 (3)	-0.01		
7 Herculis . . .	16 39.5	39 07	4.59 (4)	4.59			1.26 (4)	1.26

in the last column of table 4. In deriving this relation photographic magnitudes from 4 catalogues, and visual magnitudes from 2 catalogues were used, all reduced to the Harvard Scale.

The duality of formula (2) is probably produced by instrumental causes, or by the method of reduction; either the variety of the F_l scales used, or some residual effect of the blend of short-and long-wave radiation is responsible for the duality. For this reason it is advisable to use, instead of F_l , the reduced colour-indices J_t as given by formulae (2). The objection that there may be lost the advantage of a great interval in wavelength is not valid, as in this case the numerical value of the probable error is changed in the same proportion as the colour-equivalent itself.

As to the UR colour-equivalents, neither of the possible sources of error mentioned above could influence them, and therefore they may be used without alteration.

From the internal agreement of the data, as well as from a comparison with other sources, the values of the true probable errors given in table 5 were derived. The probable errors include 1) errors of the estimates, 2) local errors of the single images, 3) systematic errors of the plates.

Table 5. *Probable Errors.*

a) UR colour-equivalent (*violet — photographic*).

		Number of Plates			
		1	2	3	4
Average Number of Exposures per Plate		p. e., st. mg.			
	1	± 0.10	± 0.07	± 0.06	± 0.05
	2	± 0.09	± 0.06	± 0.05	± 0.04
	3	± 0.08	± 0.06	± 0.04	± 0.04
	4	± 0.08	± 0.05	± 0.04	± 0.04

b) J_t , derived from F_l according to formulae (2) (*photographic — visual*).

		Number of Plates			
		1	2	3	4
Average Number of Exposures per Plate		p. e. st. mg.			
	1	± 0.07	± 0.05	± 0.04	± 0.03
	2	± 0.06	± 0.04	± 0.04	± 0.03
	3	± 0.06	± 0.04	± 0.03	± 0.03
	4	± 0.05	± 0.04	± 0.03	± 0.03

c) Photographic Magnitude, derived from estimates of the *ring*.

Number of Plates	1	2	≥ 3
p. e. st. mg.	± 0.05	± 0.04	± 0.03

4. Discussion of Results.

Table 6 contains a summary of the data relating to our selected list of stars. The magnitudes and the colour-equivalents were derived not only from the observations made at Tartu, but other sources were also used.

The 1st column (the column of № not counted) gives the name of the star; the order is the same as in table 4.

The 2nd column gives the visual magnitude on the Harvard Scale; this is the mean of the values given by the *Harvard* and the *Potsdam* authorities, the following systematic corrections being added to the Potsdam magnitudes:

H. D. Sp.	B_0-A_3	A_5	F_0	F_2	F_5	F_8	
Correction	-0.26	-0.22	-0.19	-0.17	-0.15	-0.12	
H. D. Sp.	G_0	G_5	K_0	K_2	K_5	K_8	M
Correction	-0.10	-0.06	-0.02	0.00	+0.06	+0.08	+0.10

In parentheses is given the number of catalogues used.

The 3^d column gives the concluded photographic magnitude; the letters indicate the catalogues used:

L = Leiden, by E. Hertzsprung (*loc. cit.*);

K = E. S. King (*loc. cit.*);

G = Göttingen Actinometrie;

T = Tartu (table 4 of the present publication).

The magnitudes were reduced to a uniform scale with the aid of the following corrections, assumed according to E. Hertzsprung (except the correction of K):

correction of $L = -0.08 - 0.08i + 0.038 m_L$;

„ „ $G = -0.13 - 0.08i + 0.061 m_g$;

„ „ $K = -0.03$.

Here i denotes the preliminary value of the colour-index, which need not be known with great precision. To T no correction was applied, as the system of magnitudes is already based on the corrected magnitudes of the three preceding catalogues.

The relative weights of the catalogues were assumed as follows:

Catalogue	L	K	G	T ,	T ,
				1 plate	2 or more plates
Weight of photogr. magn. }	1	1	1	$\frac{1}{2}$	1

Table 6.

№	Name	<i>m_{vis.}</i> H. S.	<i>m_{ptgr.}</i> sources	<i>m_{p-m}</i> p. e.	<i>J_t</i> p. e.	<i>J</i>		<i>UR</i>	
						concluded	c. i. p. e.	viol.-blue	c. i. p. e.
1	β Cancri	3.75 (2)	5.29 L,G,T	1.54 $\pm$ 0.05	...	1.54 $\pm$ 0.05	...	...	...
2	α Ursae Maj.	3.40 (2)	4.37 L,K,T	0.97 .05	0.99 $\pm$ 0.06	0.98 .04	...	...	...
3	δ Cancri	4.10 (2)	5.21 L,G,T	1.11 .05	1.11 .06	1.11 .04	...	...	...
4	ϵ Cancri A	4.16 (2)	5.14 L,T	0.98 .05	0.95 .06	0.97 .04	...	...	...
5	ζ Hydrae	3.31 (2)	4.36 G,T	1.05 .05	1.06 .06	1.05 .04	...	...	...
6	ι Ursae Maj.	3.15 (2)	3.37 L,K,T	0.22 .05	0.14 .06	0.19 .04	...	...	...
7	38 Lyncis comb.	3.77 (2)	3.96 L,T	0.19 .05	0.05 .04	0.11 .03	0.29 $\pm$ 0.08	...	...
8	40 Lyncis	3.37 (2)	5.03 L,K,T	1.66 .04	1.60 .04	1.63 .03	1.24 .08	...	...
9	θ Ursae Maj.	3.32 (2)	3.75 L,K,T	0.43 .04	0.44 .06	0.43 .03	0.06 .08	...	...
10	11 Leo Min.	5.56 (2)	6.28 T	0.72 .06	0.83 .06	0.78 .04	0.12 .09	...	...
11	α Leonis	3.74 (2)	4.23 L,G,T	0.49 .04	0.53 .04	0.51 .03	0.27 .08	...	...
12	ϵ Leonis	3.12 (2)	4.02 L,T	0.90 .05	0.74 .04	0.80 .03	...	...	...
13	μ Leonis	4.08 (2)	5.42 L,T	1.34 .05	1.32 .04	1.33 .03	...	...	...
14	20 Leo Min.	5.57 (2)	6.07 T	0.50 .06	0.51 .06	0.50 .04	0.05 .09	...	...
15	η Leonis	3.57 (2)	3.62 L,T	0.05 .05	0.25 .04	0.17 .03	...	...	...
16	α Leonis	1.40 (2)	1.25 L,K,T	-0.15 .04	...	-0.15 .04	-0.27 .06	...	...
17	Grb. 1616	7.19 (2)	7.54 T	0.35 .06	-0.05 .07	0.18 .05	0.02 .10	...	...
18	Bo 7493	6.74 (2)	8.19 T	1.45 .06	1.26 .07	1.37 .05	0.87 .10	...	...
19	35 Leonis	6.02 (2)	6.67 T	0.65 .06	0.46 .05	0.54 .04	0.16 .09	...	...
20	ζ Leonis	3.60 (2)	3.86 L,K,T	0.26 .04	0.38 .03	0.32 .03	0.54 .08	...	...
21	39 Leonis	5.86 (2)	6.39 T	0.53 .06	0.40 .06	0.46 .04	-0.08 .08	...	...
22	Boss 2750	5.84 (2)	6.33 T	0.49 .06	0.49 .04	0.49 .03	0.07 .09	...	...
23	μ Ursae Maj.	3.28 (2)	4.97 L,K,T	1.69 .04	1.61 .04	1.65 .03	1.72 .08	...	...
24	Bo 7627	6.50 (2)	7.11 T	0.61 .07	0.46 .06	0.52 .05	...	...	...
25	ρ Leonis	3.78 (2)	3.78 L,T	0.00 .05	...	0.00 .05	-0.70 .05	...	...
26	38 Leo Min.	5.94 (2)	6.46 T	0.52 .06	0.83 .06	0.68 .04	0.24 .06	...	...
27	42 Leo Min.	5.35 (2)	5.21 T	-0.14 .06	...	-0.14 .06	0.18 .06	...	...
28	β Ursae Maj.	2.40 (2)	2.34 L,K,T	-0.06 .05	0.11 .06	0.01 .04	...	...	...
29	22 H Camel.	7.64 (2)	8.96 T	1.32 .06	1.64 .07	1.45 .05	0.78 .07	...	...
30	ψ Ursae Maj.	3.20 (2)	4.37 L,T	1.17 .05	1.28 .06	1.22 .04	...	...	...
31	δ Leonis	2.62 (2)	2.73 L,K,T	0.11 .04	0.26 .06	0.16 .03	0.35 .08	...	...
32	θ Leonis	3.36 (2)	3.40 L,K,T	0.04 .04	0.10 .06	0.06 .03	0.36 .06	...	...
33	ξ Ursae Maj. comb.	3.84 (2)	4.43 L,K,T	0.59 .04	0.66 .06	0.61 .03	0.04 .08	...	...
34	ν Ursae Maj.	3.66 (2)	5.17 L,T	1.51 .05	1.50 .06	1.51 .04	1.10 .08	...	...
35	90 Leonis comb.	5.82 (2)	5.82 G,T	0.00 .05	...	0.00 .05	-0.28 .06	...	...
36	Boss 3078	5.78 (2)	6.18 T	0.40 .06	0.29 .06	0.34 .04	-0.11 .08	...	...
37	ν Virginis	4.25 (2)	5.82 L,G,T	1.57 .04	1.70 .06	1.61 .03	1.32 .08	...	...
38	93 Leonis	4.58 (2)	5.11 L,T	0.53 .05	0.48 .06	0.51 .04	0.31 .08	...	...
39	β Leonis	2.30 (2)	2.28 L,K,T	-0.02 .05	0.40 .06	0.16 .04	...	...	...
40	β Virgin.	3.77 (2)	4.19 G,K,T	0.42 .04	0.48 .06	0.44 .03	0.15 .08	...	...
41	Lund 5297	6.48 (2)	7.19 T	0.71 .06	0.75 .06	0.73 .04	0.00 .09	...	...
42	γ Ursae Maj.	2.48 (2)	2.41 L,K,T	-0.07 .04	...	-0.07 .04	0.27 .08	...	...
43	Nic. 3352	...	9.13 T	...	0.67 .07	0.67 .07	...	...	...
44	δ Urs. Maj.	3.35 (2)	3.45 L,K,T	0.10 .04	0.01 .06	0.07 .03	0.13 .08	...	...
45	Bo 8468	...	10.96 T	...	1.25 .07	1.25 .07	0.68 .10	...	...
46	Ci 1553	...	10.26 T	...	0.44 .10	0.44 .10	-0.42 .10	...	...
47	β Can. Ven.	4.34 (2)	4.91 L,K,T	0.57 .04	0.59 .06	0.58 .03	0.01 .08	...	...
48	Boss 3294	5.87 (2)	7.56 G,T	1.69 .05	1.44 .06	1.58 .04	1.08 .10	...	...
49	γ Virg. comb.	2.90 (1)	3.14 K,T	0.24 .06	0.46 .06	0.35 .04	...	...	...
50	ϵ Ursae Maj.	1.79 (2)	1.79 L,K,T	0.00 .04	0.00 .06	0.00 .03	0.22 .08	...	...
51	δ Virgin.	3.70 (2)	5.19 G,K,T	1.49 .04	1.58 .04	1.54 .03	1.32 .08	...	...
52	12 Can. Ven. A	2.92 (2)	2.78 L,K,T	-0.14 .04	...	-0.14 .04	-0.07 .04	...	...

Table 6.

8		9		10		11		12		13		14		15		16	
No.	Spectrum		Absol. magn. spectrosc.	μ	Classification of Abs. Magn. from												
	HD.	Mt. W.			Colour and Mt. W. Sp.		b μ	$\frac{a+b}{2}$	Colour, μ and HD. Sp.	Colour and μ , or Colour only							
1	K ₂	K ₄	0.0 (3)	7".4	(0.5)	0.1	0.4	0.2	0.7	g	0.3	gK ₄					
2	G ₀	G ₂	-0.6 (2)	16.	(-0.6)	-1.5	1.2	-0.2	-0.8	g	-0.3	gG ₆					
3	K ₀	G ₉	0.5 (4)	24.	(0.8)	0.6	2.4	1.5	1.0	g	0.8	gG ₉					
4	G ₅	G ₅	-1.2 (2)	5.4	(-0.4)	-0.9	0.2	-0.4	-0.4	g	-0.3	gG ₆					
5	K ₀	G ₅	-0.8 (2)	10.	(-0.4)	-1.9	0.5	-0.7	1.0	g	0.4	gG ₈					
6	A ₅	A ₄ n	2.2 (3)	50.	...	...	2.6	...	2.2	d	2.2	dA ₅					
7	A ₂	B ₉ n	0.6 (1)	14.	...	...	1.4	...	1.5	d	1.7	dA ₃					
8	K ₅	K ₅	0.7 (3)	21.	(0.4)	0.9	1.7	1.3	0.4	g	0.4	gK ₅					
9	F ₈ p	F ₅	3.0 (2)	109.	(3.2)	2.8	3.9	3.4	3.6	d	3.2	dF ₅					
10	K ₀	K ₁	5.3 (3)	75.	(6.1)	6.6	4.7	5.6	6.0	d	6.0	dK ₀					
11	F ₅ +A ₃	F ₅ p	1) -1.4 (1)	15.	(-1.)	-1.0	1.4	0.2	-1.	g	-1.	gF ₅					
12	G ₀ p	G ₀	-0.9 (2)	4.6	(-0.8)	-0.4	-0.7	-0.6	-0.8	g	-0.8	gG ₀					
13	K ₀	K ₃	0.9 (4)	24.	(0.6)	0.6	2.4	1.5	1.0	g	0.6	gK ₃					
14	G ₅	G ₇	5.2 (2)	68.	(5.0)	6.0	4.6	5.3	4.4	d	3.6	dF ₈					
15	A ₀ p	A ₂	1) ...	1.2	...	...	-2.2	...	-2.	g	-2.	gA					
16	B ₈	B ₈ n	0.4 (1)	25.	...	...	0.7	...	0.5	d	-0.5	dB ₅					
17	F ₂	...	...	10.	...	...	3.0	...	3.1	d	2.7	dF ₀					
18	K ₅ p	K ₆	8.1 (2)	145.	(8.2)	7.5	6.4	7.0	7.6	d	10.	dM					
19	G ₀	G ₁	3.3 (1)	21.	(4.0)	4.0	3.3	3.6	3.9	d	4.4	dG ₅					
20	F ₀	F ₀	0.1 (2)	2.5	(-1.)	-0.3	-1.2	-0.8	-1.	g	-1.	gF ₃					
21	F ₅	F ₈	3.7 (1)	47.	(3.6)	4.9	4.2	4.6	3.2	d	3.3	dF ₆					
22	F ₅	F ₉	3.5 (1)	19.	(3.8)	4.3	3.1	3.7	3.2	d	3.6	dF ₈					
23	K ₅	K ₅	0.7 (3)	8.2	(0.4)	-1.1	0.2	-0.4	0.4	g	0.1	gM					
24	G ₀	G ₂	5.6 (1)	90.	(4.1)	4.1	5.6	4.8	3.9	d	3.8	dF ₉					
25	B ₀ p	...	-2.9 (1)	1.0	...	...	-2.3	...	-2.	...	-1.	B					
26	G ₅ p	F ₉	2.2 (1)	23.	(-0.8)	0.6	3.4	2.0	4.4	d	4.4	dG ₅					
27	B ₉	B ₉ n	0.8 (1)	4.8	...	...	0.9	...	0.7	d	0.5	dB ₈					
28	A ₀	A ₃ s	1.0 (1)	8.8	...	...	-0.1	...	1.0	d	1.2	dA ₁					
29	Mb	Ma	10.5 (1)	478.	(10.)	8.0	8.7	8.4	10.2	d	10.	dM					
30	K ₀	K ₀	1.3 (3)	6.9	(1.0)	0.5	-0.2	0.2	1.0	g	0.7	gK ₂					
31	A ₃	A ₂ n	1.4 (2)	21.	...	...	1.2	...	1.7	d	2.0	dA ₄					
32	A ₀	A ₂ s	0.8 (2)	10.	...	...	0.6	...	1.0	d	1.5	dA ₂					
33	G ₀	F ₉ +G ₂	2) 5.0 (2)	73.	2) (3.8)	3.4	3.6	3.5	3.9	d	3.9	dG ₀					
34	K ₀	K ₄	0.3 (2)	2.7	(0.4)	0.1	-0.9	-0.4	1.0	g	0.5	gK ₄					
35	B ₃	...	-1.8 (1)	1.2	...	...	-0.8	...	-1.	...	-0.5	B ₅					
36	F ₅	F ₂	3.7 (2)	35.	(3.1)	3.7	3.9	3.8	3.2	d	3.1	dF ₃					
37	Ma	Ma	0.0 (3)	19.	(0.1)	0.2	2.1	1.2	0.1	g	0.1	gM					
38	F ₈	F ₄	2.6 (2)	16.	(-1.)	-1.7	1.9	0.1	-1.	g	-1.	gF ₅					
39	A ₂	A ₂ n	1.5 (2)	51.	...	...	2.0	...	1.5	d	2.0	dA ₄					
40	F ₈	F ₇	3.5 (2)	79.	(3.4)	4.3	3.8	4.0	3.6	d	3.2	dF ₅					
41	G ₅	G ₈	6.0 (1)	705.	(5.3)	5.8	8.4	7.1	4.4	d	6.0	dK ₀					
42	A ₀	A ₀ n	0.9 (1)	9.4	...	...	0.0	...	1.0	d	0.7	dB ₀					
43	G ₅	G ₈	6.4 (1)	53.	(5.3)	5.3	6.1	5.7	4.4	d	5.3	dG ₈					
44	A ₂	A ₀ n	0.9 (1)	11.	...	...	0.6	...	1.5	d	0.7	dB ₉					
45	K ₀	Ma	8.5 (1)	57.	(10.)	10.6	7.0	8.8	7.6	dK ₅	7.6	dK ₅					
46	G ₅	...	...	30.	...	...	6.4	...	4.4	d	5.±	blue dwarf (Fe)					
47	G ₀	G ₀	4.4 (2)	76.	(3.9)	3.9	4.0	4.0	3.9	d	3.9	dG ₀					
48	Ma	Mb	0.7 (1)	9.0	(0.1)	2.1	2.0	2.0	0.1	g	0.4	gK ₅					
49	F ₀ +F ₀	F ₀ +F ₂	2) 2.6 (2)	56.	...	...	2.6	...	2.7	d	3.2	dF ₄					
50	A ₀ p	A ₂ s	0.6 (2)	11.	...	...	-0.3	...	1.0	d	1.0	dA ₀					
51	Ma	Mb	0.3 (3)	48.	(0.1)	1.3	3.0	2.2	0.1	g	0.4	gK ₅					
52	A ₀ p	A ₁ s	0.6 (1)	23.	...	...	1.4	...	1.0	d	0.0	dB ₇					

1) c-characteristic of Miss Maury. 2) Mean.

Table 6. Continued.

1		2		3		4		5		6		7	
№	N a m e	<i>m</i> _{vis.}	<i>m</i> _{ptgr.}	<i>m</i> _{p-mv}	p. e.	<i>J</i> _t	p. e.	<i>J</i>		<i>UR</i>			
		H. S.	sources					concluded	p. e.	viol.-blue	p. e.		
								c. i.		c. i.			
53	ε Virgin.	3.02 (2)	3.93 L,K,T	0.91	+0.04	0.97	+0.03	0.95	+0.02	0.33	+0.06		
54	14 Can. Ven.	5.14 (2)	5.12 T	-0.02	.06			-0.02	.06	0.16	.06		
55	BD.+23° 2537	6.74 (2)	7.16 T	0.42	.06	0.20	.04	0.27	.03	0.11	.10		
56	Boss 3398	5.92 (2)	7.62 T	1.70	.06	1.50	.04	1.57	.03	1.11	.10		
57	43 Comae	4.34 (2)	4.92 L,K,T	0.58	.04	0.55	.06	0.57	.03	0.08	.08		
58	Be A 4874	6.53 (2)	7.62 G,T	1.09	.05	0.88	.06	1.00	.04	0.47	.08		
59	64 Virgin.	5.86 (2)	6.18 G,T	0.32	.05	0.14	.06	0.24	.04	0.27	.08		
60	ζ U. Maj. A.	2.38 (2)	2.27 L,K,T	-0.11	.04			-0.11	.04	0.24	.08		
61	γ Ursae Maj.	3.99 (2)	4.20 L,T	0.21	.05			0.21	.05	0.28	.08		
62	70 Virgin.	5.16 (2)	5.86 G,T	0.70	.05	0.51	.06	0.62	.04	0.16	.06		
63	ζ Virgin.	3.37 (2)	3.54 K,T	0.17	.05	0.13	.06	0.15	.04	0.47	.08		
64	Be A 5003		9.86 T			1.34	.07	1.34	.07	0.89	.10		
65	τ Bootis	4.55 (2)	5.00 L,G,K,T	0.45	.04			0.45	.04	0.09	.06		
66	η Ursae Maj.	1.96 (2)	1.66 L,K,T	-0.30	.04			-0.30	.04	-0.36	.05		
67	ν Bootis	4.20 (2)	5.83 L,G,T	1.63	.04	1.58	.03	1.60	.02	1.25	.06		
68	η Bootis	2.89 (2)	3.29 L,K,T	0.40	.04	0.52	.03	0.48	.02	0.20	.04		
69	α Bootis	0.23 (2)	1.26 L,K,T	1.03	.04	1.34	.06	1.13	.03	0.70	.06		
70	Lal. 26 294		9.83 T			1.28	.07	1.28	.07				
71	θ Bootis	4.16 (2)	4.59 L,T	0.43	.05	0.44	.06	0.43	.04	0.06	.08		
72	ρ Bootis	3.76 (2)	5.06 L,T	1.30	.05	1.11	.04	1.19	.03	0.82	.04		
73	γ Bootis	3.08 (2)	3.22 L,K,T	0.14	.04	0.21	.03	0.19	.02	0.34	.04		
74	ζ Bootis	3.82 (2)	3.92 L,T	0.10	.05	0.08	.03	0.09	.02	0.34	.06		
75	ε Boot. 1) comb.	2.50 (2)	3.44 L,K,T	0.94	.04	0.96	.03	0.95	.02	0.42	.08		
76	109 Virg.	3.75 (2)	3.86 T	0.11	.06	0.01	.03	0.03	.03	0.30	.05		
77	β Bootis	3.64 (2)	4.55 L,K,T	0.91	.04	0.97	.03	0.95	.02	0.50	.06		
78	Boss 3867	5.95 (2)	7.64 G,T	1.69	.05	1.17	.06	1.47	.04	1.18	.10		
79	BD.+19° 2937	7.38 (2)	9.05 (G),T	1.67	.06			1.67	.06	1.11	.10		
80	BD.-0° 2943		8.85 T							0.30	.10		
81	Nic. 3891 comb.	6.66 (1)	7.36 T	0.70	.07	0.81	.06	0.77	.05	0.26	.10		
82	δ Bootis	3.56 (2)	4.45 L,T	0.89	.05			0.89	.05	0.41	.04		
83	50 Bootis	5.37 (2)	5.28 T	-0.09	.07			-0.09	.07	0.16	.08		
84	η Coronae comb.	5.10 (2)	5.60 T	0.50	.07			0.50	.07	0.01	.08		
85	γ Ursae Min.	3.11 (2)	3.14 L,K,T	0.03	.05	0.01	.06	0.02	.04				
86	β Coronae	3.72 (2)	3.93 L,T	0.21	.05	0.19	.03	0.20	.03	0.17	.04		
87	θ Coronae	4.18 (2)	3.97 L,T	-0.21	.05			-0.21	.05	-0.19	.06		
88	α Coronae	2.32 (2)	2.15 L,K,T	-0.17	.04			-0.17	.04	0.33	.04		
89	χ Serpentis	5.25 (2)	5.51 G,T	0.26	.05			0.26	.05	0.28	.08		
90	γ Coronae	3.82 (2)	3.82 L,T	0.00	.05	-0.15	.06	-0.06	.04	0.11	.08		
91	β Serpentis	3.66 (2)	3.66 L,T	0.00	.05	-0.01	.03	-0.01	.03	0.37	.05		
92	κ Serpentis	4.22 (2)	6.06 L,G,T	1.84	.04			1.84	.04	1.52	.08		
93	μ Serpentis	3.63 (1)				-0.01	.06	-0.01	.06				
94	ε Serpentis	3.72 (2)								0.31	.08		
95	χ Herculis	4.64 (2)	5.25 L,T	0.61	.06			0.61	.06	-0.02	.08		
96	γ Serpentis	3.89 (2)	4.31 L,G,T	0.42	.04	0.39	.04	0.40	.03	0.06	.04		
97	Lal. 29307	7.18 (2)	7.82 T	0.64	.06	0.92	.0	0.75	.05	0.51	.10		
98	τ Herculis	3.91 (2)	3.64 L,T	-0.27	.05			-0.27	.05	-0.36	.06		
99	γ Herculis	3.78 (2)	3.97 L,T	0.19	.05	0.17	.04	0.18	.03	0.44	.04		
100	λ Ophiuchi	3.78 (2)	3.88 T	0.10	.06	0.11	.06	0.10	.04	0.33	.08		
101	β Herculis	2.90 (2)	3.81 L,K,T	0.91	.05			0.91	.05	0.17	.08		
102	12 Ophiuchi	5.87 (1)	6.38 T	0.51	.07	0.73	.06	0.64	.05	0.45	.10		
103	ζ Herculis	3.03 (2)	3.50 L,K,T	0.47	.05			0.47	.05	-0.01	.08		
104	η Herculis	3.68 (2)	4.55 L,K,T	0.87	.05	0.84	.06	0.86	.04				

1) Difference of magnitude 2.4 vis..

Table 6. Continued.

8		9		10		11		12		13		14		15		16	
№	Spectrum		Absol. magn. spectrosc.	μ	Classification of Abs. Magn. from												
	HD.	Mt. W.			^a Colour and Mt. W. Sp.		^b μ	$\frac{a+b}{2}$	^c Colour, μ and HD. Sp.	^d Colour and μ , or Colour only							
53	K ₀	G ₆	0.2 (2)	27."	(-0.2)	0.4	1.8	1.1	1.0 g	-0.4	gG ₅						
54	B ₉	B ₉ n	0.6 (1)	2.6	...	...	-0.1	...	0.7 d	0.7	dB ₉						
55	F ₀	...	...	...	...	...	...	...	2.7 d	...	...						
56	Mb	Mc	-1.0 (1)	6.5	(0.1)	2.1	1.4	1.8	0.1 g	0.5	gK ₄						
57	G ₀	G ₁	4.7 (2)	118.	(4.0)	4.0	4.7	4.4	3.9 d	4.2	dG ₈						
58	K ₀	K ₃	6.4 (1)	69.	(6.4)	6.4	5.1	5.8	6.1 dK ₁	6.4	dK ₃						
59	A ₀	A ₅ s	1.2 (2)	8.0	...	...	1.8	...	1.0 d	2.3	dA ₇						
60	A ₂ p	A ₂ s	1.1 (1)	15.	...	...	0.4	...	1.5 d	0.7	dB ₉						
61	A ₅	...	2.7...	15.	...	...	1.8	...	2.2 d	2.2	dA ₆						
62	G ₀	G ₃	4.0 (2)	63.	(4.2)	3.2	4.2	3.7	3.9 d	5.0	dG ₇						
63	A ₂	...	...	29.	...	...	2.1	...	1.5 d	2.0	dA ₄						
64	K ₂	Mb	10.2 (1)	230.	(10.)	8.8	8.3	8.6	7.6 dK ₅	10.	dM						
65	F ₅	F ₇	3.0 (2)	47.	(3.4)	3.8	3.5	3.6	3.2 d	3.3	dF ₆						
66	B ₃	...	-0.8 (1)	12.	...	...	0.0	...	-1. d	-0.7	B ₄						
67	K ₅	...	...	10.	...	...	1.1	...	0.4 g	0.1	gM						
68	G ₀	F ₉	2.6 (2)	37.	(3.8)	2.9	2.0	2.4	3.9 d	3.4	dF ₇						
69	K ₀	K ₀	1.0 (4)	228.	(1.0)	1.0	2.9	2.0	1.0 g	1.0	gK ₀						
70	...	K ₆	8.4 (1)	73.	(8.2)	8.9	6.6	7.8	...	7.6	dK ₅						
71	F ₈	F ₆	3.0 (1)	48.	(3.3)	3.9	3.3	3.6	3.6 d	3.2	dF ₅						
72	K ₀	K ₃	0.4 (3)	15.	(0.6)	0.2	1.7	1.0	1.0 g	0.7	gK ₂						
73	F ₀	A ₇	1.8 (2)	18.	...	...	1.2	...	2.2 dA ₅	2.2	dA ₅						
74	A ₂	A ₀ n	0.9 (1)	6.1	...	...	0.2	...	1.5 d	1.5	dA ₂						
75	K ₀ +A ₀	G ₈ +A ₂ n	²⁾ -0.8 (2)	4.9	(-1.)	1.9	-1.0	0.4	-1. sg	-0.4	gG ₅						
76	A ₀	...	0.6 (1)	11.	...	...	1.1	...	1.0 d	1.2	dA ₁						
77	G ₅	G ₄	-0.6 (2)	6.3	(-0.5)	-1.9	0.0	-0.8	-0.4 g	-0.3	gG ₆						
78	Mb	Mb	0.2 (1)	0.5	(0.1)	4.0	-2.0	1.0	0.1 g	0.5	gK ₄						
79	K ₂	...	...	8.0	...	...	2.9	...	0.7 g	0.4	gK ₅						
80	K ₀	...	...	32.	...	...	5.2	...	6.0 d	6.0	dK ₀						
81	K ₀	K ₀	5.8 (1)	138.	(6.0)	5.5	6.4	6.0	6.0 d	6.0	dK ₀						
82	K ₀	G ₅	-0.6 (1)	16.	(-0.4)	-0.4	1.3	0.4	-0.4 gG ₅	-0.6	gG ₃						
83	B ₉	A ₀ n	0.7 (1)	5.0	...	...	1.0	...	0.7 d	0.5	dB ₈						
84	G ₀	G ₀	4.2 (2)	24.	(3.9)	4.5	3.0	3.8	3.9 d	3.6	dF ₈						
85	A ₂	...	1.2 (1)	1.7	...	...	-2.1	...	-2. g	-2.	gA ₁						
86	F ₀ p	F ₂	-0.3 (1)	19.	(2.7)	1.3	1.7	1.5	-1. g	2.2	dA ₆						
87	B ₅	...	...	3.7	...	...	-0.3	...	-0.5 d	-0.5	B ₅						
88	A ₀	A ₀ n	0.9 (2)	16.	...	...	0.5	...	1.0 d	0.7	dB ₉						
89	A ₀ p	A ₂ ns	1.1 (2)	4.4	...	...	0.5	...	1.0 d	2.4	dA ₈						
90	A ₀	A ₀ n	1.1 (1)	10.	...	...	0.9	...	1.0 d	0.5	dB ₈						
91	A ₂	A ₀ n	1.1 (1)	9.1	...	...	0.7	...	1.5 d	1.0	dA ₀						
92	K ₅	Ma	0.1 (2)	11.	(0.1)	-5.8	1.2	-2.3	0.4 g	0.1	gM						
93	A ₀	A ₀ sn	0.4 (1)	9.5	...	...	0.7	...	1.0 d	1.0	dA ₀						
94	A ₂	A ₅ s	1.8 (1)	13.	...	...	1.2	...	1.5 d	1.7	dA ₃						
95	G ₀	F ₇	4.1 (2)	76.	(3.4)	2.5	4.1	3.3	3.9 d	3.9	dG ₀						
96	F ₅	F ₅	4.2 (2)	133.	(3.2)	3.2	4.5	3.8	3.2 d	3.2	dF ₅						
97	G ₀	K ₀	5.7 (1)	86.	(6.0)	4.5	6.0	5.2	6.0 dK ₀	6.1	dK ₁						
98	B ₅	...	-0.5 (1)	3.2	...	...	-0.7	...	-0.5 d	-0.7	dB ₄						
99	F ₀	A ₂	1.4 (2)	6.2	...	...	0.2	...	2.2 dA ₅	2.0	dA ₄						
100	A ₀	A ₁ n	1.2 (2)	9.7	...	...	0.9	...	1.0 d	1.7	dA ₃						
101	K ₀	G ₅	-0.8 (2)	11.	(-0.4)	0.6	0.3	0.4	1.0 g	-0.7	gG ₂						
102	K ₀	K ₀	5.6 (1)	54.	(6.0)	6.0	4.5	5.2	6.0 d	5.6	dG ₉						
103	G ₀	G ₁	2.7 (2)	60.	(4.0)	4.9	2.8	3.8	3.9 d	3.3	dF ₆						
104	K ₀	G ₅	1.0 (2)	10.	(-0.4)	0.6	0.8	0.7	-0.4 gG ₅	-0.8	gG ₁						

2) c-characteristic of Miss Maury.

The 4th column gives the difference: *photographic* — *visual* magnitude, or the colour-index, with its probable error; the probable error is given by the following table:

Table 7. *Probable error of colour-index.*

	Number of photographic catalogues			
	1/2 ¹⁾	1	1 1/2--2 1/2	≥3
	p. e.			
1 visual catalogue	±0.08	±0.07	±0.06	±0.06
2 " "	±0.07	±0.06	±0.05	±0.04

A comparison with section b) of table 5 suggests the superiority of Tikhoff's method, as compared with the ordinary method of determining colour-indices: one image on a single plate gives with Tikhoff's method the colour-index with the same accuracy as the visual and photographic magnitudes, derived each from 1 *catalogue*.

The 5th column gives J_t , the colour-index derived from the *Fl* plates according to Tikhoff's method; J_t was computed according to formulae (2) from the data of the last column of table 4; the probable error is taken from table 5, section b).

The 6th column gives the concluded value of the colour-index, $J = \text{photographic} - \text{visual}$ magnitude, and its probable error; this value is the mean of the data given in the two preceding columns; in computing the mean the weights were assumed in accordance with the probable errors.

The 7th column gives the *UR* colour-equivalent = *violet* — *photographic* magnitude, and the corresponding probable error.

Next follows the spectrum, according to the *Henry Draper* Catalogue, (8th column) and according to the *Mount Wilson* classification (9th column).

The 10th column gives the spectroscopic absolute magnitude; the data of this column are taken from different sources; mean values of the data of different catalogues are simply assumed, the number of catalogues used being given in parentheses. The absolute magnitudes used were from the *Mount Wilson Observatory*²⁾,

1) A magnitude derived at Tartu from only 1 plate is assumed equivalent to 1/2 a catalogue.

2) By W. S. Adams and his collaborators.

Table 8.

Number of stars given in parentheses.

a) Dwarfs

Mean Sp. (Mt. W.) . . .	$M_{a,b}$	K_6	K_3	K_0	G_5	G_0	F_6	F_2	A_5	A_2	A_0	B_9	B_5
Mean Abs. Magn. Spectrosc. ($\pi = 0''.1$) . . .	9.7	8.2	6.4	6.0	4.4	3.9	3.3	3.1	1.9	1.0	0.8	0.7	-0.7
Mean Colour-Index (photogr.—vis.) . . .	1.35(3)	1.32(2)	1.00(1)	0.72(6)	0.56(2)	0.54(10)	0.46(8)	0.29(3)	0.20(5)	0.08(9)	-0.01(10)	-0.02(3)	-0.19(5)
Mean UR Colour-Equival. (viol.—photogr.) . . .	0.78(3)	0.87(1)	0.47(1)	0.27(5)	0.10(2)	0.09(9)	0.08(8)	-0.04(2)	0.33(5)	0.32(6)	0.27(9)	0.21(3)	-0.29(5)

b) Giants

Mean Sp. (Mt. W.) . . .	$M_{a,b,c}$	K_5	K_3	K_0	G_5	G_1	F_5	F_1	A_2
Mean Abs. Magn. Spectrosc. ($\pi = 0''.1$) . . .	0.1	0.4	0.6	1.0	-0.4	-0.8	-1.4	-0.1	(e-characteristic)
Mean Colour-Index (photogr.—vis.) . . .	1.60(6)	1.59(5)	1.26(2)	1.15(3)	0.94(7)	0.89(2)	0.51(1)	0.26(2)	0.17(1)
Mean UR Colour-Equival. (viol.—photogr.) . . .	1.26(6)	1.33(4)	0.82(1)	0.70(1)	0.35(4)	. . .	0.27(1)	0.36(2)	. . .

In this table are not included the following peculiar stars: $\iota 2$ Can. Ven., of Sp. $A_1 p$, is too rich in violet light for this spectrum; the colour is better accounted for by Sp. B_7 or B_8 ;

ϵ Bootis, Sp. $K_0 + A_0$; ρ Leonis, Sp. $B_0 p$.

For a few stars the HD spectrum was assumed, these stars being not classified at Mount Wilson.

the *Victoria Astrophysical Observatory*¹⁾, the *Norman Lockyer Observatory*²⁾, and the *Arcetri Observatory*³⁾.

The 11th column gives μ , the centennial proper motion.

The last 5 columns (12 to 16) contain the result of an attempt to classify the stars according to absolute magnitude, using the colour-equivalents together with the spectrum or the proper motion. The classification is of an exploratory character; the small number of stars does not allow of determining the precise character of the relation of colour to absolute magnitude and spectrum. Nevertheless, even the results of our rough classification indicate that the colour of the stars may be used with success in determining their absolute magnitude, with an accuracy sufficient at least for statistical purposes. The principles of the classification will be briefly explained below.

a) *Colour, absolute magnitude and spectrum.* Table 8 contains mean values of the colour-equivalents of the stars of table 6, grouped according to absolute magnitude and spectrum.

The data of table 8 are represented graphically on fig. 1, 2 and 3. The curves for the giants and the dwarfs are clearly separated: on fig. 1 (colour-index and spectrum) from F_5 to M ; on fig. 2 (UR colour-equivalent and spectrum) from F_0 to M , data for giants (supergiants) earlier than F_0 being not available; on fig. 3 (colour-index and UR colour-equivalent) from c. $i + 0.25$ to $+0.60$. The data, though scarce, are of such a character that single stars agree well with the mean of the groups. The curve for UR (fig. 2 and 3) is of an irregular character, due to the absorption in the violet region of the spectrum; the absorption changes with spectral type and is especially strong for $A_0 - A_5$.

Our results are in general agreement with the results of Seares (Mt. Wilson Comm. to Nat. Acad., № 59, 1919), except for the M -stars. For this class we find a perceptible difference in colour of giants and dwarfs, whereas Seares does not find such a difference. It may be remarked that Seares applied systematic corrections to the colour derived from single plates; also the *Purkinje effect* must have been a troublesome source of error in the method of exposure-ratios, especially in the case of the faint M -dwarfs. In our method no systematic differences of colour were found for

1) By R. K. Young and W. E. Harper.

2) By W. J. S. Lockyer, D. L. Edwards and W. B. Rimmer.

3) By G. Abetti.

the single plates, and the Purkinje phenomenon plays no part in the reduction; we therefore feel confident of our results. Of course, the number of *M*-dwarfs used in both cases is very small; 4 by Seares, and 3 in the present investigation; but the smallness of the number does hardly account for the different results obtained.

b) Fig. 1 and 2 may be used for determining *absolute magnitudes* from the colour and spectrum, or the colour and proper motion. The absolute magnitudes may be derived in two different ways: α) by attempting to determine the luminosity of each star individually; β) by discerning only between the *giant* and the *dwarf* branches of stellar luminosities, and by assigning to stars of a given branch and spectrum the same mean absolute magnitude. These two methods we shall afterwards denote briefly by α and β . Which method is to be preferred depends upon whether the mean error of α is greater or smaller than the dispersion of the true luminosities of stars around their mean value assumed in β .

1) *Colour and Mount Wilson spectrum*. Method α was attempted by simply assuming a linear relation between colour and absolute magnitude. Both colour-equivalents were used, with weights depending upon the ratio of the probable error to the distance between the curves on fig. 1 or 2 respectively. Only spectra from F_5 (colour-index) or F_0 (*UR* colour-equivalent) and later have been classified, as for the earlier spectra the data regarding the colour of giants are not available, and the absolute magnitudes would represent simply a repetition of the Mount Wilson absolute magnitudes. The result is contained in the 12th column of table 6. For comparison are given in parentheses in the same column absolute magnitudes derived from the same data, but according to method β .

2) *Colour, Mount Wilson spectrum and proper motion*. The absolute magnitude determined from the proper motion and apparent magnitude according to Kapteyn's formula

$$M_{abs.} = m_{vis.} + 5 + 5 \log \pi,$$

$$\log \pi = -0.690 - 0.0713m + 0.645 \log \mu^1)$$

is given in the 13th column of table 6. As the sets of absolute

1) Mount Wilson Contributions 188. For large μ and m the formula gives too small values of π .

magnitudes in the 12th and 13th columns are approximately of equal weight, their simple mean is given in the 14th column. The absolute magnitudes in this column are individual, i. e. correspond to method α . With method β the absolute magnitudes become exactly the same as those given in column 12 in parentheses, except 38 Leo Minoris, for which the absolute magnitude becomes 3.8, instead of -0.8 .

3) *Colour and HD spectrum.* The results are bad, which must be attributed to the smaller accuracy of the *HD* classification; about 20% of the giants are classified as dwarfs, and *vice versa*; the results, as being of little use, are not included in table 6.

4) *Colour, HD spectrum and proper motion.* Only method β was applied. The results are given in the 15th column of table 6; the meaning of the letters is

$d = \text{dwarf}; g = \text{giant};$

the corresponding absolute magnitude precedes the letter. In a few cases the *HD* required some correction; in these cases the new adopted spectrum is given after the letter d or g ; the new spectrum was chosen as near as possible to the spectrum of *HD*, but so that the colour would not conflict with the *dwarf* or *giant* classification adopted.

5) *Colour and proper motion only, method β .* The subdivision into giants and dwarfs is made from the proper motion, in a few cases checked by the colour; the effective spectral type is derived from the two colour-equivalents. The method gives, besides a mean value of the absolute magnitude, an independent spectral classification, the accuracy of which depends upon the slope of the curve representing the colour-equivalent as a function of the spectrum. The result is given in the 16th (last) column of table 6; in this column the concluded spectrum is preceded by the letters d (dwarf) or g (giant) and the corresponding adopted absolute magnitude.

The probable errors of the absolute magnitudes derived according to the different methods are given below. These probable errors require some correction for the probable error of the spectroscopic luminosities which were used as a standard of comparison; the correction is, however, small and may be neglected.

<i>Probable Errors of Absolute Magnitude, derived from:</i>		
method α , colour and Mt. W. spectrum,	p. e. = ± 1.04 st. mg.	
" " Kapteyn's formula (m and μ),	p. e. = ± 0.80 " "	
" " colour, Mt. W. sp., proper		
motion and m ,	p. e. = ± 0.70 " "	
method β , colour and Mt. W. spectrum,	p. e. = ± 0.63 " "	
" " colour, Mt. W. sp., proper		
motion and m ,	p. e. = ± 0.59 " "	
" " colour, proper motion, m ,		
and H. D. sp.	p. e. = ± 0.61 " "	
" " colour, proper motion and m only	p. e. = ± 0.62 " "	

Method β leads thus to better results; this indicates that the dispersion of luminosities of stars of a given spectrum and branch is smaller than the errors of method α ¹⁾.

One of the possible causes, and probably the chief one which affects the accuracy of colour-parallaxes is the existence of composite spectra; double stars were not excluded from our observing list, because the intention was to obtain such a material as may be assumed representative of a statistical investigation relating to faint stars.

c) *Effective temperatures.* Fig. 4 represents the values of $\frac{c_2}{T}$ according to Wilsing ²⁾, plotted against the colour-indices of column 6, table 6. There are in all 55 stars in common to our table and Wilsing's list. The relation between $\frac{c_2}{T}$ and J is well represented by the following linear relation:

$$\frac{c_2}{T} = 1.49 + 1.839 J \dots (3).$$

It may be assumed that the effective temperatures of individual stars derived from the colour-index are more accurate than the temperatures given by Wilsing.

On the other hand, the *UR* colour-equivalents cannot be used for deriving effective temperatures on account of the variable absorption in the violet region of the spectrum, a circumstance already pointed out by Wilsing. Table 9 gives the effective

1) The failure of method α may also be partly due to the non-linear relation of colour to absolute magnitude; such a non-linear relation was found by Seares, and it also seems to be confirmed by some of our data.

2) *Potsdam Publ.* № 76 (1919).

temperature for different groups of stars of our list, together with the surface brightness and the hypothetical density; in deriving the densities the masses were assumed according to A. S. Eddington's theory¹).

Table 9.

D w a r f s					G i a n t s				
Sp. (Mt.W.)	n	$T_{abs.}^{2)}$	Visual sur- face bright- ness, st. mg.	Hypothetical Density, $\odot = 1$	Sp.	n	$T_{abs.}^{2)}$	Vis. surf. br., st. mg.	Hypothet. Density, $\odot = 1$
M	3	3640 ⁰	+3.12	6.3	M	6	3300 ⁰	+3.93	0.00007
K_6	2	3730	+2.94	1.40	K_5	5	3310	+3.89	0.0001
K_3	1	4380	+1.80	0.87	K_3	2	3830	+2.72	0.0004
K_0	6	5180	+0.80	1.52	K_0	3	4060	+2.32	0.0009
G_5	2	5790	+0.22	0.53	G_5	7	4530	+1.58	0.0005
G_0	10	5890	+0.14	0.35	G_1	2	4670	+1.41	0.0004
F_6	8	6240	-0.14	0.22	F_5	1	6010	+0.04	0.0014
F_2	3	7230	-0.77	0.45	F_1	2	7410	-0.87	0.016
A_5	5	7850	-1.09	0.18					
A_2	9	8960	-1.56	0.13					
A_0	10	9930	-1.89	0.16					
B_9	3	10100	-1.93	0.15					
B_5	5	12800	-2.55	0.07					

5. Summary.

1. Two kinds of colour-equivalents derived from photometric observations in three different spectral regions are determined for a selected list of stars.

2. The colour-equivalents, used together with the proper motion and apparent magnitude, determine the absolute magnitude with a probable error of less than ± 0.6 st. mg. The accuracy may probably be increased when a larger material will be available, i. e. when it will be possible to determine more precisely the relation of colour to absolute magnitude and spectrum.

1) *Monthly Notices* 84 p. 308—332 (1924).

2) $c_2 = 14600$.

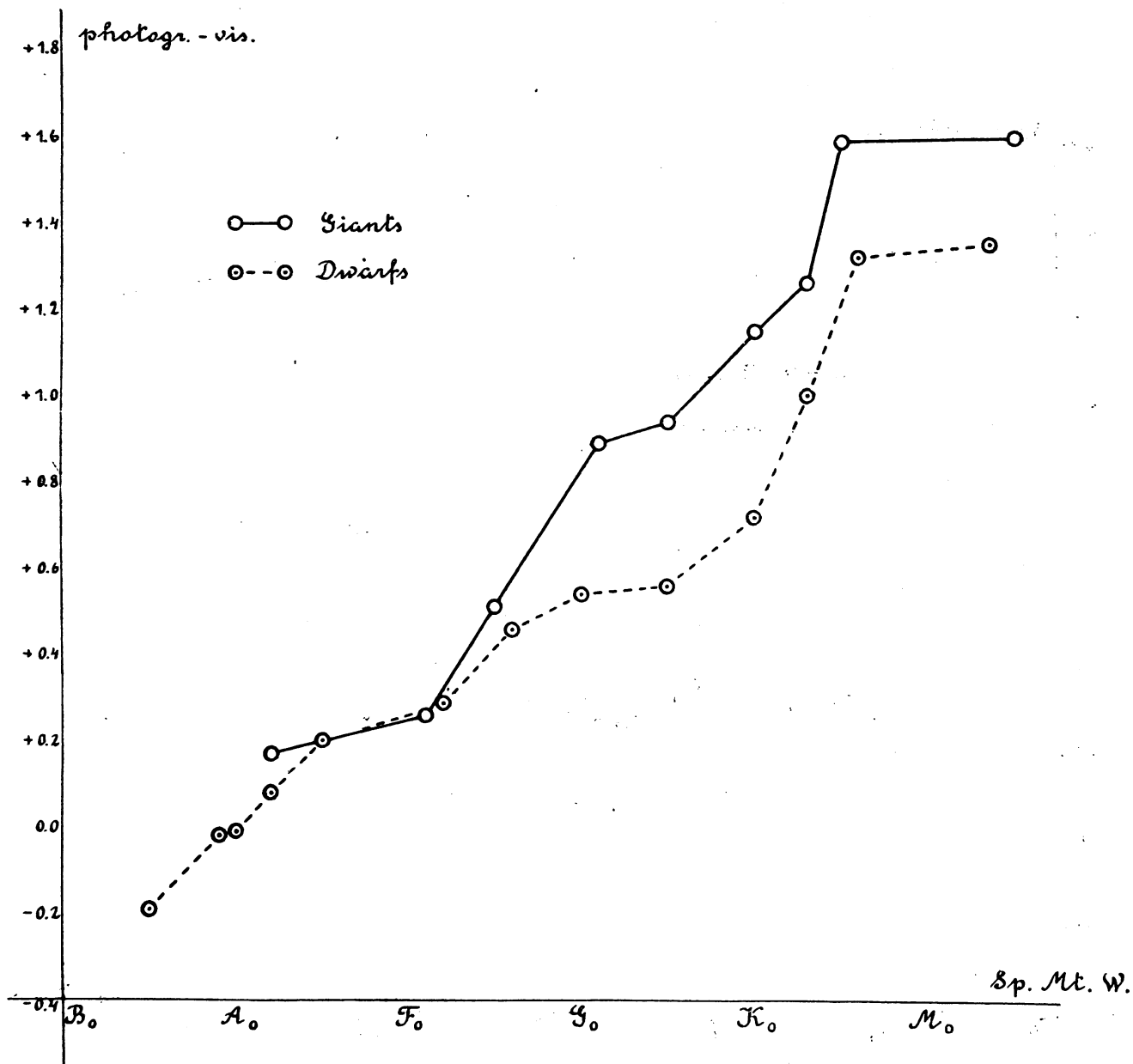


Fig. 1.

T. P. 263

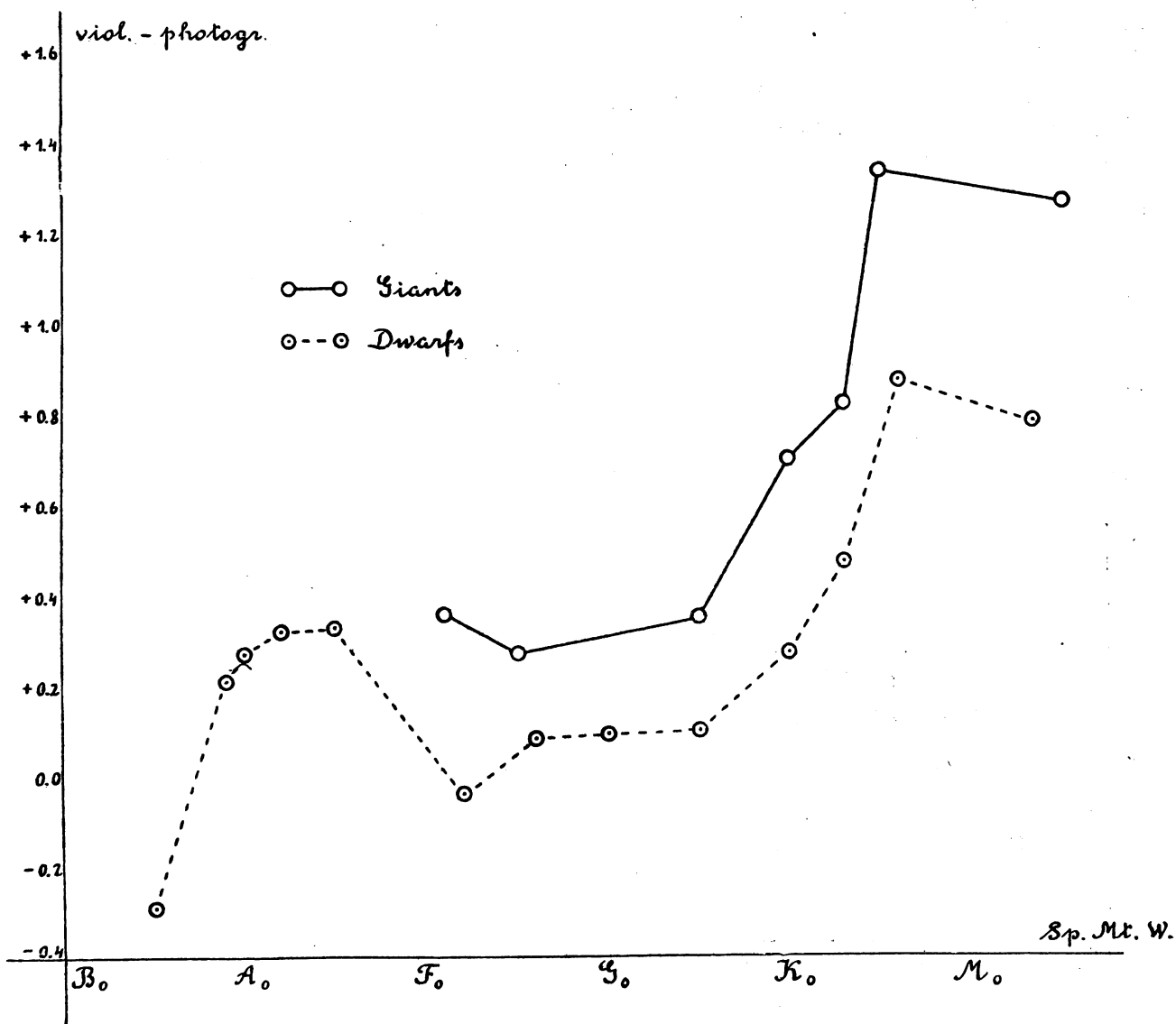


Fig. 2.

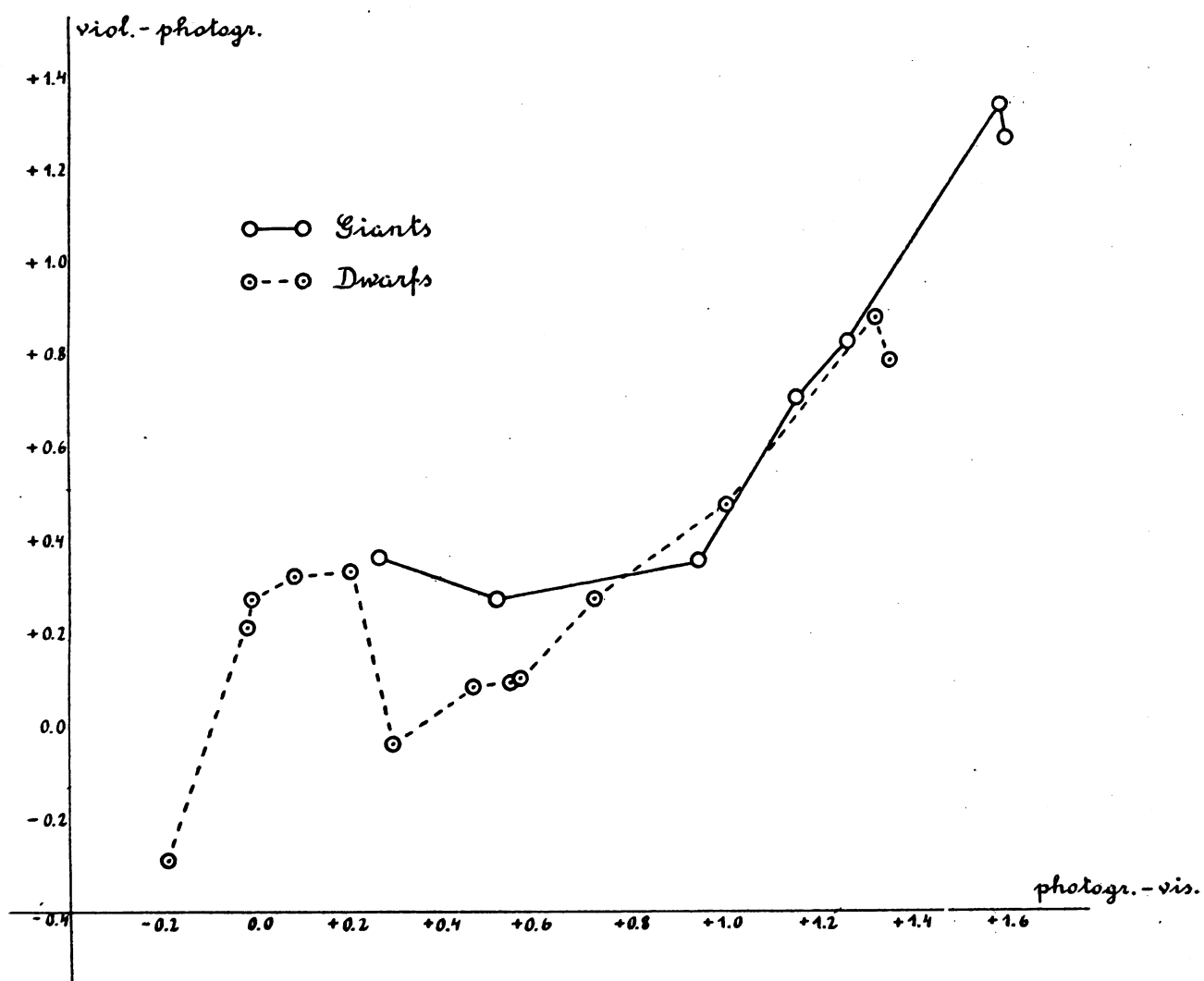


Fig. 3.

T. P. 263

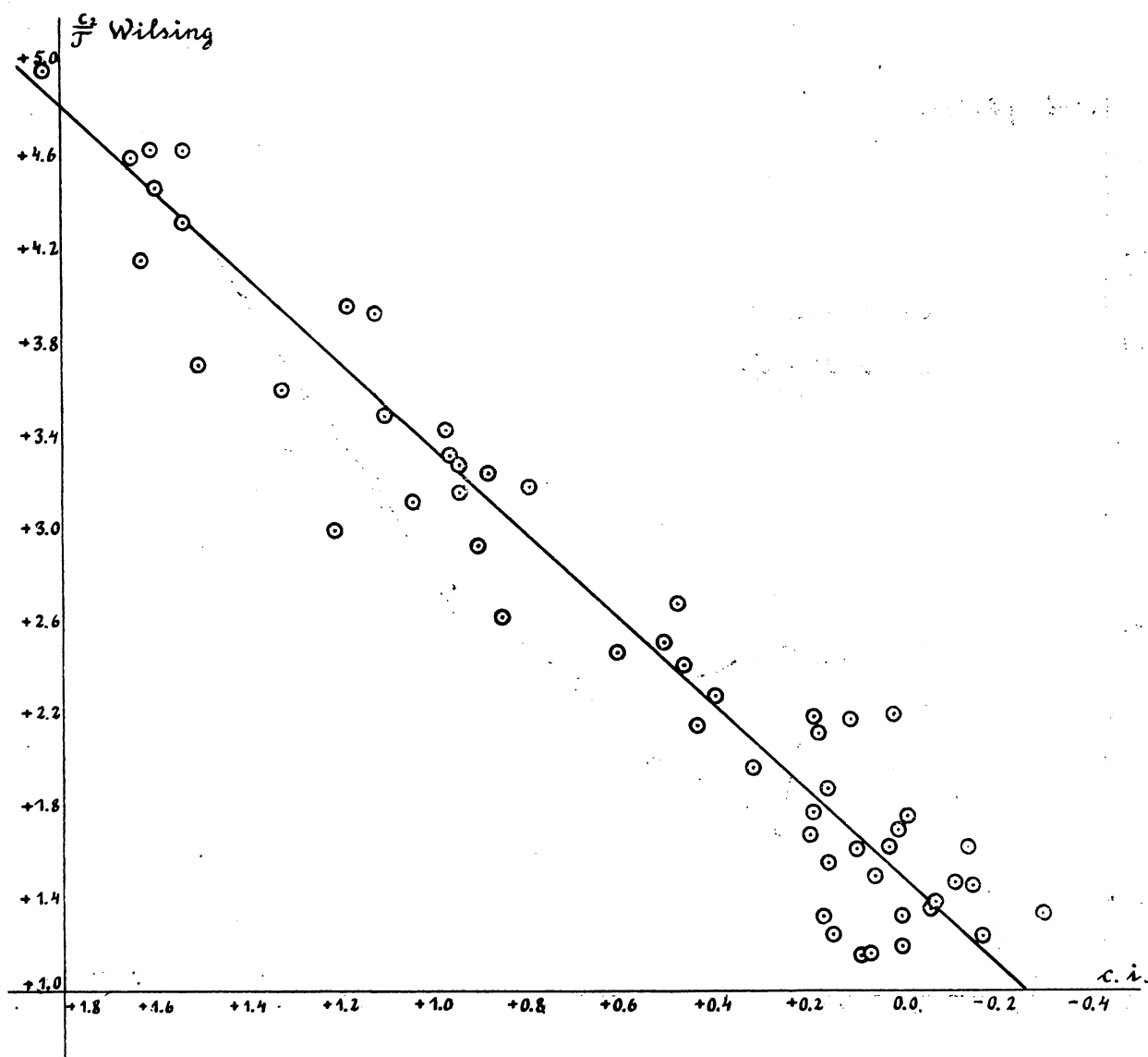


Fig. 4.