Multiple Stars and Equipartition of Energy

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1. The apparent equipartition of kinetic energy is a conspic-uos observational fact of our stellar universe. Although satisfied, at least qualitatively, over a wide range of stellar masses and under special conditions of selection ($cf.$[1]), the law $mv^2 = $ const. cannot yet be considered as universally valid; there exist whole classes of stars for which a larger velocity seems to be connected with a larger mass (compare main sequence A stars with late type giants). If group motion may be responsible for some of these anomalies, the fact of the mere existence of group motion indicates that our present stellar universe is younger than the "time of relaxation", and that equipartition of energy by mutual approaches of the stars cannot yet have taken place ($cf.$[2]). Thus, if some form of equipartition nevertheless exists, the laws of the kinetic theory of a gas in a steady state cannot account for it. The equipartition may reflect conditions that prevailed at an early stage of our stellar universe. Building up of large masses from small ones moving at random leads also to equipartition of energy as a consequence of the fundamental laws of probability. If there exists a certain stratification in the Galaxy, of such a character that different classes of stars are generated at different places, the relative velocity of a star in the neighbourhood of the sun must depend upon its point of origin: the latter defines the elements of the galactic orbit, which in their turn determine the differential velocity of the star, relative to the average galactic rotational velocity around us. For distant stars the direct difference in galactic rotation may be responsible for certain anomalies (O stars).

To throw more light upon the problem, more varied statistical material is required. Below the correlation of the velocity of the star with its degree of multiplicity is investigated. In comparing double or multiple stars with single ones of the same spectrum and absolute magnitude, new conditions of testing the equipartition of energy are introduced.

2. Our results below are based upon radial velocities [3] of the stars of the Preliminary General Catalogue by B. Boss, thus

practically upon radial velocities of naked eye stars. Owing to peculiar selection it did not seem advisable to use radial velocities of fainter stars; also, for these the data referring to multiplicity are less complete than for the brighter stars. The radial velocities were freed from the solar ·motion, assuming conventionally the following elements of the latter: apex, $a = 270^0$, $\delta = + 34^0$; $v_{\odot} = 20$ km/sec. With these elements charts by Smart[4] could be used.

Three major groups of absolute magnitude were distinguished: the main sequence stars, the giants, the supergiants. The separation of main sequence stars from giants is chiefly based upon the Yale catalogue[5]. For stars without parallax determinations the magnitude — proper motion equivalent of the absolute magnitude was used as a criterion; the danger of selecting in such a manner systematically slow giants and fast dwarfs is apparently of no practical importance in the present case. The supergiants include all c stars, as well as Cepheid variables and K and M stars brighter than absolute magnitude — 1.5 contained in the lists of Payne[6], Merrill[7], and Mount Wilson[8].

Four classes of multiplicity were distinguished: a) single stars; b) visual binaries with faint companions, $\Delta m \geqslant 2.5$, and spectroscopic binaries with one spectrum visible; c) visual binaries with bright companions, $\Delta m \leqslant 2.4$, and spectroscopic binaries with two spectra visible; d) multiple stars. The classification of multiplicity was made according to Öpik[9], Moore[10,11], and according to remarks found in different other catalogues (Boss, Schlesinger, Mount Wilson). Stars with composite H. D. spectra were counted with the spectroscopic binaries having two spectra visible.

Tables 1, 2 and 3 contain the result; $\bar{\varrho}$ is the arithmetical mean radial velocity, corrected for solar motion and taken without regard to sign ; n is the number of stars. The probable error in $\bar{\varrho}$ is computed from the approximate formula

$$p.\,e. = \pm \frac{\varrho}{2\sqrt{n}} \quad \ldots \ldots \ldots \quad (1).$$

In Table 1, spectra $K0 - M$ are omitted because the material is too scarce. In Table 3, the two classes of binaries are joined together.

Table 1.

Main Sequence Stars.

H. D. Spectrum	a) Single		b) Vis. bin. with Δm $\geqslant$ 2.5; spec. bin. with one sp.		c) Vis. bin. with Δm $\leqslant$ 2.4; sp. bin. with two sp.		d) Multiple	
	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n
O	34.8 ± 6.2	8	28.8 ± 5.1	8	20.0 ± 10.0	1	14.4 ± 2.5	8
B0 — B2	14.2 ± 1.0	51	15.3 ± 1.6	22	9.2 ± 1.3	13	10.2 ± 1.0	24
B3 — B5	8.6 ± 0.3	222	8.8 ± 0.4	120	10.9 ± 0.9	40	8.2 ± 0.7	40
B8 — B9	10.4 ± 0.4	169	11.5 ± 0.6	81	11.7 ± 1.0	32	7.7 ± 0.8	22
A0 — A2	11.0 ± 0.3	353	11.5 ± 0.4	166	11.5 ± 0.6	83	11.0 ± 0.7	57
A3 — A5	12.7 ± 0.5	138	13.5 ± 0.9	51	16.0 ± 1.4	35	10.0 ± 1.1	19
F0 — F2	14.6 ± 0.6	153	14.8 ± 1.0	53	13.4 ± 1.1	36	11.5 ± 1.1	28
F5 — F8	17.4 ± 0.7	168	14.4 ± 0.9	62	17.2 ± 1.5	35	13.7 ± 1.3	26
G0 — G5	28.2 ± 1.3	119	22.8 ± 2.3	24	18.9 ± 1.9	26	19.4 ± 2.7	13
All	13.6 ± 0.2	1381	12.6 ± 0.3	587	13.4 ± 0.4	301	11.0 ± 0.4	237

Table 2.

Giant Stars.

H. D. Spectrum	a) Single		b) Vis. bin. with Δm $\geqslant$ 2.5; spec. bin. with one sp.		c) Vis. bin. with Δm $\leqslant$ 2.4; sp. bin. with two sp.		d) Multiple	
	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n
G0 — G5	16.9 ± 0.6	175	13.5 ± 1.0	48	10.4 ± 1.1	21	12.6 ± 2.0	10
K0 — K2	17.2 ± 0.3	731	18.9 ± 0.7	159	16.8 ± 1.8	21	13.8 ± 1.4	23
K5 — Ma	19.4 ± 0.6	270	17.1 ± 1.4	37	9.2 ± 2.3	4	23.8 ± 4.9	6
Mb — Md	20.2 ± 1.3	62	18.4 ± 2 9	10	20.5 ± 7.3	2	8.5 ± 5.4	2
All	17.8 ± 0.3	1238	17.6 ± 0.6	254	13.4 ± 1.0	48	14.7 ± 1.1	41

An increasing degree of multiplicity indicates doubtlessly an increasing mass: therefore, from the standpoint of equipartition of energy, we expect a decreasing velocity with increasing multiplicity. The figures of Tables 1, 2 and 3 agree generally with the expected trend, but conspicuous exceptions occur also; thus, the well-represented class A0 — A2 of the main sequence (Table 1) does not reveal any definite change of velocity with multiplicity.

Table 3.

Supergiant Stars.

H. D. Spect-rum	a) Single		b) + c) Binary		d) Multiple	
	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n	$\bar{\varrho}$ p. e.	n
B	16.0 ± 1.6	26	14.5 ± 1.8	16	7.2 ± 1.5	6
A	18.6 ± 2.5	14	17.5 ± 3.1	8	1. ± 5.	1
F	10.5 ± 0.9	31	13.8 ± 1.7	17	9.0 ± 1.7	7
G	8.9 ± 1.2	14	9.0 ± 1.2	14	10.2 ± 1.8	8
K0 — K2	9.2 ± 1.1	17	7.9 ± 1.1	12	9.5 ± 3.4	2
K5 — Ma	5.5 ± 0.7	14	7.0 ± 1.1	11	35. ± 17.5	1
All	11.7 ± 0.5	116	11.6 ± 0.7	78	9.7 ± 1.0	25

Further, in those cases where a decrease in $\bar{\varrho}$ with increasing multiplicity occurs, the effect appears to be too small as compared with the expected change in mass. Below we try to analyse the correlation more precisely. Although the actual mass ratios for stars of a different degree of multiplicity are unknown, a more or less reliable estimate of these ratios can be made. We have to consider that few stars which are classified as single are truly single: most of them are likely to have non-observable companions, the mass of which may equal a sensible fraction of the mass of the primary (cf.[9]). Taking into account the mass-luminosity relation, the probable frequency of invisible companions [9], and the average mass ratio of non-observable companions [12], we estimate the average mass ratios of the different classes of multiplicity as follows:

	a) Single	b) Visual binaries with $\varDelta$ m ⩾ 2.5; spectroscopic binaries with one spectrum visible	c) Vis. binaries with $\varDelta$ m < 2.5; sp. bin. with two spectra visible	d) Multiple
$\dfrac{\text{Mass of system}}{\text{Mass of primary}}$	1.2	1.47	1.90	2.6

In the case of true equipartition of energy we should have

$$\bar{\varrho} = C\mu^{-\frac{1}{2}} \quad \ldots \ldots \ldots \quad (2),$$

where μ is the mass, and C a constant. For our purposes it is convenient to assume $\mu = 1$ for group a) of multiplicity, and to transcribe (2) as follows

$$\varrho = C - Cx,$$

where $x = 1 - \mu^{-\frac{1}{2}}$. For $\mu = 1$, $x = 0$, $\varrho = C$; for $\mu = \infty$, $x = 1$, $\bar{\varrho} = 0$. When equipartition of energy is not fulfilled strictly, the effective correlation may be generally written in a different form:

$$\bar{\varrho} = C_0 - C_1 x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

The degree of approximation to equipartition is characterized by the ratio $\dfrac{C_1}{C_0}$, which may be called the "coefficient of equipartition". When $\dfrac{C_1}{C_0} = 1$, true equipartition of energy takes place; $0 < \dfrac{C_1}{C_0} < 1$ indicates "partial equipartition" — a decrease of velocity with increasing mass which is slower than required by (2); $\dfrac{C_1}{C_0} \leqslant 0$ indicates absence of equipartition.

From the above estimated relative masses we derive the following, slightly smoothed values of x:

	a)	b)	c)	d)	b) + c)
$x = 1 - \mu^{-\frac{1}{2}}$	0.00	0.11	0.21	0.31	0.13

With these adopted values of x, least-square solutions of (3) were obtained from the data of Tables 1, 2, and 3. The result is contained in Table 4. We notice that our scale of x is comparatively insensible to errors in the adopted mass ratios; if, nevertheless, a scale error in x exists, its influence would consist in yielding systematically too large, or too small values of C_1 and of the ratio $\dfrac{C_1}{C_0}$. The possible extreme values of the mass ratio d):a) may be estimated to lie between 1.7 and 3.0, which gives for d) the extreme values of x from 0.23 to 0.42, as compared with 0.31 adopted; the extreme systematic error in the scale of $\dfrac{C_1}{C_0}$ is therefore from — 26 to + 35 per cent; the probable error of the scale is perhaps ± 10 per cent.

The probable errors in Table 4 are computed on the basis of the a p r i o r i errors quoted in Tables 1, 2, and 3; the actual deviations from correlation (3) were not taken into account.

Table 4.

Correlation of Velocity and Mass of System, for Same Spectrum and Luminosity (least-square solutions)

Spectrum	C_0 p. e.	C_1 p. e.	$\dfrac{C_1}{C_0}$ p. e.
Main Sequence			
O	34.7 ± 5.0	64.2 ± 18.6	+ 1.85 ± 0.54
$B0 - B2$	14.6 ± 1.0	14.1 ± 4.5.	+ 0.97 ± 0.31
$B3 - B5$	8.6 ± 0.3	— 1.8 ± 2.3	— 0.21 ± 0.27
$B8 - B9$	11.1 ± 0.4	4.6 ± 2.8	+ 0.41 ± 0.25
$A0 - A2$	11.2 ± 0.3	0.0 ± 2.2	0.00 ± 0.20
$A3 - A5$	13.0 ± 0.5	0.0 ± 3.9	0.00 ± 0.30
$F0 - F2$	15.0 ± 0.6	8.6 ± 3 8	+ 0.57 ± 0.25
$F5 - F8$	16.9 ± 0.6	10.0 ± 4.6	+ 0.59 ± 0.27
$G0 - G5$	28.0 ± 1.2	36.0 ± 8.4	+ 1.29 ± 0.30
$O - G5$	13.6 ± 0.2	6.2 ± 1.4	+ 0.46 ± 0.10
Giants			
$G0 - G5$	16.7 ± 0.6	24.3 ± 5.0	+ 1.46 ± 0.30
$K0 - K2$	17.0 ± 0.3	— 3.0 ± 4.0	— 0.18 ± 0.24
$K5 - Ma$	19.5 ± 0.6	— 1.3 ± 10.9	— 0.07 ± 0.56
$Mb - Md$	19.8 ± 1.3	22.9 ± 15.0	+ 1.16 ± 0.76
$G0 - Md$	17.8 ± 0.3	10.6 ± 2.9	+ 0.60 ± 0.16
Supergiants			
B	16.4 ± 1.5	25.3 ± 8.5	+ 1.54 ± 0.52
A	20.0 ± 2.0	32.3 ± 17.0	+ 1.62 ± 0.85
F	11.1 ± 0.9	0.0 ± 5.7	0.00 ± 0.51
G	8.8 ± 1.0	— 3.2 ± 6.4	— 0.36 ± 0.73
$K0 - K2$	9.3 ± 1.0	7.6 ± 6.2	+ 0.82 ± 0.67
$K5 - Ma$	(4.4 ± 1.3)	(— 53.0 ± 26.0)	(— 12. ± 6.)
$B - Ma$	11.6 ± 0.5	5.0 ± 3.6	+ 0.43 ± 0.31

From all data of Table 4 we find the general mean $\dfrac{C_1}{C_0} = + 0.50 \pm 0.08$; the mean values for the three major groups — the main sequence, the giants, and the sugergiants — deviate

from this general mean by less than the corresponding probable errors. Thus, for multiple systems in general the apparent equipartition of energy is incomplete; the change of velocity with the combined mass of the system amounts, on the average, to only about one half of the change required by the law of equipartition.

A closer inspection of the figures for the separate spectral classes reveals differences in $\frac{C_1}{C_0}$ which are much greater than admissible for accidental fluctuations. The data for the main sequence show a remarkable regular variation with spectral type, as represented on Fig. 1; the value of $\frac{C_1}{C_0}$ drops down to zero near spectrum $A0$ and rises on both sides of that; the observed minimum at $B3 - B5$ does not fit into the smooth curve and may be accidental. On the other hand, for the main sequence the variation of $\frac{C_1}{C_0}$ shows a remarkable similarity with the variation of C_0, the mean peculiar velocity of "single" stars (cf. Fig. 1); the unsmoothed values show even a better agreement than the smoothed values. It appears highly probable that in the present case the variation of the "coefficient of equipartition",
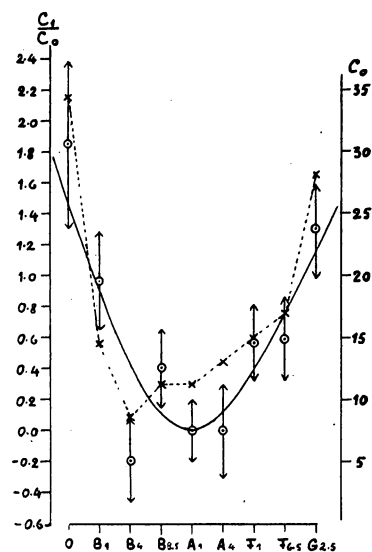

Fig. 1. Full line and dotted circles with arrows: variation of $\frac{C_1}{C_0}$ for the main sequence. Dotted line and crosses: variation of C_0 for the main sequence.

$\frac{C_1}{C_0}$, the variation of the mean peculiar velocity (C_0 for "single" stars, also $\bar{\varrho}$ for all systems, cf. Tables 5 — 8 below), and the apparent variation of the solar velocity with spectral type (cf. [13], Table I) have all one common cause, connected perhaps in some way with star streaming. The question is too complicated to be settled without a special study. Without claiming to give a true explanation, we may exempli gratia propose the following scheme which might be useful in guiding further research: let us assume that all spectral classes (of the main sequence) contain a certain fraction of objects belonging to an aggregate a of invariable kinematical properties, mixed with aggregates β of kinematic properties that

vary with spectral class; if a is rich in double and multiple stars, whereas β is poor in such systems, the observed correlation of C_0 and $\dfrac{C_1}{C_0}$ must result. Indeed, let us imagine that group β possesses a large mean peculiar velocity; in such a case the mean velocity of "single" systems increases in a greater proportion, as compared with the double or multiple systems, because upon our assumption the high velocity β stars are predominantly single, whereas double or multiple systems belong mostly to a of constant mean velocity; the result is a steeper decrease of velocity with increasing multiplicity; hence simultaneously a larger value of C_0 is accompanied by a larger value of $\dfrac{C_1}{C_0}$, and vice versa.

3. As a by-product of our computations the correlation of mean peculiar velocity with spectrum is obtained; the results, separately for the major luminosity classes, are contained in Tables 5, 6, 7 and 8. The mean radial velocity, $\bar{\varrho}$, freed from the effect of solar motion, is given; the cosmical probable error is not given, but may be easily calculated from (1). Of course, the results depend upon certain assumptions regarding the coordinates of the solar apex and upon the solar velocity, taken the same for all the spectral classes ($cf.$[4] and above); but small variations (with spectral type) in the assumed characteristics of the apex have little influence upon $\bar{\varrho}$.

Since there are too few late type dwarfs among the Boss P. G. C. stars, to the main sequence list were added dwarfs of Mt. Wilson spectrum $G5$ to $M5$ of known absolute magnitude and radial velocity. Thus, in the corresponding part of table 5 some effect of selection by large proper motion, thus by large peculiar velocity may be suspected; the admirably smooth run of the figures seems to indicate, however, that the effect of selection is negligible.

The most remarkable feature of our tables is the smooth continuous variation of $\bar{\varrho}$ with spectral type. Along the main sequence $\bar{\varrho}$ changes in the same manner as found above for C_0: it increases steadily on both sides of a minimum at $B5$; whereas the increase of $\bar{\varrho}$ along the late type branch fits well, qualitatively at least, into the concept of equipartition of energy, the increase of $\bar{\varrho}$ for spectra earlier than $B5$ is at variance with equipartition;

Table 5.

Mean Peculiar
Velocity, Main
Sequence

Sp.	$\bar{\varrho}$ km/sec	n
$0a - 0d$	39.0	4
$0e$	38.7	3
$0e5$	20.6	18
$B0$	14.3	29
$B1$	13.1	28
$B2$	12.2	53
$B3$	9.8	268
$B5$	7.1	154
$B8$	10.4	173
$B9$	11.0	131
$A0$	10.9	410
$A2$	11.8	249
$A3$	12.5	115
$A5$	13.6	128
$F0$	14.1	201
$F2$	14.2	69
$F5$	15.1	198
$F8$	19.1	93
$G0$	24.4	116
$G5 - G6$	24.6	55
$G7 - G9$	25.5	57
$K0 - K2$	26.2	76
$K3 - K5$	28.8	64
$K6 - M1$	29.2	81
$M2 - M5$	30.6	25

Table 6.

Mean Peculiar
Velocity, Giants

H. D. sp.	$\bar{\varrho}$ km/sec	n
$G0$	13.3	39
$G5$	16.0	215
$K0$	17.2	793
$K2$	18.6	141
$K5$	18.7	175
Ma	19.5	142
$Mb - Md$	19.7	76

Table 7.

Mean Peculiar
Velocity, Super-
giants

H. D. sp.	$\bar{\varrho}$ km/sec	n
B	14.4	48
A	17.5	23
F	11.3	55
G	9.2	36
$K0 - K2$	8.7	31
$K5 - Ma$	7.3	26

Table 8.

Comparison of Mean Peculiar Velocity
for Different Luminosity Groups

Sp.	$\bar{\varrho}$ km/sec		
	Main Sequence	Giants	Super-giants
B	10.0	—	14.4
A	11.7	—	17.5
F	15.3	—	11.3
G	25.5	15.6	9.2
$K0 - K2$	26.2	17.4	8.7
$K5 - Ma$	29.2	19.1	7.3

galactic rotation, which comes into play at the great distances of
these luminous early type stars, may be advocated as an adequate
explanation. However, galactic rotation apparently cannot account
for the whole effect in all its complicatedness; above we found
a simultaneous increase of the "coefficient of equipartition", $\dfrac{C_1}{C_0}$,
with C_0, or with $\bar{\varrho}$; now, the multiple systems possess a greater
intergrated luminosity and, for a given apparent limiting magni-
tude, are observed at a greater average distance than the "single"
stars; hence the effect of galactic rotation must be stronger

for the multiple systems, increasing their peculiar velocity and reducing the apparent value of $\dfrac{C_1}{C_0}$, working thus with respect to $\dfrac{C_1}{C_0}$ in a direction opposite to the observed effect.

    Tartu,

March 29, 1939.

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