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The Densities
of
Visual Binary Stars

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The Densities of Visual Binary Stars.

On the basis of certain relations between the orbital elements, and the surface brightness (estimated from the spectral type), densities of visual binaries have been computed by Öpik¹, and later by Bernevitz². At present the data referring to the orbital elements of double stars, and especially our knowledge of the relations between spectra, luminosities, and colours of the stars is much more complete than at the time of these early investigations; there are also numerous direct determinations of stellar colours at our disposal. Thus it appeared advisable to repeat the investigation, on the basis of the more complete data available at present.

1. The Methods.

Let ϱ denote the density, μ the mass, R the radius of a star; we have

$$\frac{\varrho}{\varrho_{\odot}} = \frac{\mu}{\mu_{\odot}} \cdot \left(\frac{R_{\odot}}{R}\right)^3 \dots \dots \dots (1).$$

By setting $\varrho_{\odot} = 1$, $\mu_{\odot} = 1$, $R_{\odot} = 1$, we get

$$\varrho = \frac{\mu}{R^3} \dots \dots \dots (2).$$

For a binary

$\mu_1 = \frac{a^3}{\pi^3 \cdot P^2 (1+k)} \dots \dots \dots (3)$; a denotes the semi-major axis of the orbit in seconds of arc, π — the parallax, P — the period of revolution in years; $k = \frac{\mu_2}{\mu_1}$ is the mass ratio.

For any star we have

$$R = \sqrt{\frac{I}{I_{\odot}}} \cdot \frac{E_{\odot}}{E} \dots \dots \dots (4),$$

$$\frac{I}{I_{\odot}} = \frac{206265^2}{\pi^2} \cdot 2,512^{m_{\odot}-m} \dots \dots \dots (5), \text{ and}$$

$$\frac{E_{\odot}}{E} = \left(\frac{c_2}{e^{\lambda T} - 1} \right) : \left(\frac{c_2}{e^{\lambda T_{\odot}} - 1} \right) \dots \dots \dots (6);$$

m is the apparent visual magnitude; $m_{\odot} = -26.72$; $c_2 = 14300$;

¹ E. Öpik, *Ap. J.*, 44, 292, 1916.

² E. Bernevitz, *A. N.*, 213, 1, 1921.

$\lambda = 0.56 \mu$ is the sensitivity maximum of the eye; T is the effective temperature of the star; $T_{\odot} = 5810^{\circ}$ (assumed).

The best individual values of T can be got from colour indices, standardized with the aid of spectrophotometric temperature determinations; values of T derived in such a manner are more accurate than the spectrophotometric values themselves³. On the basis of Öpik's data³ we assume

$$\frac{C_2}{T} = 1.47 C + 1.82 \dots \dots (7),$$

where C denotes the colour index in a special system⁴.

Neglecting the member — 1 in Planck's formula (for the actual range of temperatures considered here this is permissible), equations (2) to (7) give us:

$$\log \varrho_1 = \log \frac{a^3}{P^2} - \log (1 + k) + 0.6 m_1 - 1.71 C_1 + 0.83 \dots \dots (8),$$

$$\log \varrho_2 = \log \frac{a^3}{P^2} - \log \left(1 + \frac{1}{k}\right) + 0.6 m_2 - 1.71 C_2 + 0.83 \dots \dots (8a);$$

ϱ_1 and ϱ_2 denote the densities, m_1 and m_2 the apparent visual magnitudes, C_1 and C_2 the colours (in the system chosen here⁴) of the two components of a binary.

When no direct determinations of colour were available, the values of C in (8), (8a) were estimated on the basis of spectrum and luminosity*. A linear relationship between absolute magnitude and colour was assumed. In estimating the colours, for stars earlier than $F0$, H. D. spectra were used when available; for later stars, the more accurate Mt. Wilson spectra were preferred⁵.

In the case of B and A stars, the surface brightness as inferred from the colour is probably underestimated, partly on account of Balmer absorption, partly by other reasons⁶. Therefore the true densities of A stars are probably slightly greater than the computed densities.

³ E. Öpik, *Ap. J.*, **81**, 177, 1935.

⁴ E. Öpik, *Publ. Tartu Obs.*, **27**, 1, 1929. The visual magnitude was assumed equal to the mean of Harvard and Potsdam.

* cf.³, Table II; also ⁴, Tables 37, 38.

⁵ Walter S. Adams, Alfred H. Joy, Milton L. Humason, and Ada Margaret Brayton, *Ap. J.*, **81**, 187, 1935; *Mt. W. Contr.*, Nr. **511**.

⁶ E. Öpik, *Harv. Circ.*, Nr. **359**, 1931.

2. The *Ti O* Effect.

A more serious trouble arises in the case of *M* stars on account of absorption by *Ti O* bands, which affect considerably the visual magnitude, and only slightly the photographic magnitude; the colour appears thus "bluer" than it would be without *Ti O* absorption; therefore the surface brightness as based on the apparent colour is overestimated. Fortunately, it seems to be possible to take into account the *Ti O* effect in quite a satisfactory manner.

As the photographic magnitude (m_p) is less influenced by *Ti O* absorption than the visual magnitude, we may calculate the true effective temperature of a star from

$$m_p - m_b = \frac{36100}{T} + 10 \log T - 43.40 \dots \dots \dots (9).$$

The bolometric magnitude, m_b , we assume according to Pettit and Nicholson⁷; for m_p we take a mean of different determinations, reduced to a certain system⁴. The constant term of (9) is determined from *K* stars of Pettit and Nicholson's list, in which case the *Ti O* effect is probably negligible, and the temperature is given by (7).

After having found the "true" temperature of an *M* star from (9), the "ideal" colour, C' , can be computed from (7) by substituting in this equation for T the "true" temperature; the *Ti O* correction is then $\Delta C = C' - C$. Table I contains the results. The spectra in this table are from Mt. Wilson⁵.

As shown by the data, the *Ti O* effect increases rapidly with the advancing *M* spectrum. Fig. 1 represents the dependence of the average ΔC upon spectrum.

Evidently the same correction ΔC must be applied both to the colour, and (with opposite sign) to the apparent magnitude:

$$\left. \begin{aligned} C' &= C + \Delta C \\ m' &= m - \Delta C \end{aligned} \right\} \dots \dots \dots (10).$$

For *M* stars these corrected (still undercorrected) values of C' and m' were substituted into (8), (8 a), in calculating the densities; the smoothed mean values of ΔC , as represented by Fig. 1, were actually used.

⁷ E. Pettit and S. Nicholson, *Ap. J.*, **68**, 279, 1928; *Mt. W. Contr.*, Nr. **369**.

Table I.
The *TiO* Correction of Colour, ΔC .

Star	Sp.	m_p	m_b	T	C'	C	ΔC	$\overline{\Delta C}$
β And	M0	4.08	0.87	3075	1.92	1.74	+0.18	
Cin 1218	M0	8.17	5.78	3480	1.56	1.36	0.20	
61 ² Cyg	M0	7.70	5.31	3480	1.55	1.52	0.03	
α Sco	M1	3.06	-0.96	2780	2.26	2.11	0.15	
α Ceti	M2	4.66	1.13	2950	2.06	1.80	0.26	
α Ori	M2	2.69	-1.31	2790	2.25	1.98	0.27	
π Leo	M2	6.64	3.40	3060	1.94	1.82	0.12	
Boss 2935	M2	9.04	5.80	3060	1.94	1.45	0.49	0.25(6)
β Peg	M2	4.27	0.65	2920	2.09	1.80	0.29	
55 Peg	M2	6.40	3.40	3170	1.83	1.74	0.09	
η Gem	M3	5.39	1.56	2850	2.17	1.70	0.47	
μ Gem	M3	4.85	1.02	2850	2.17	1.73	0.44	0.48(3)
δ Vir	M3	5.24	1.59	2900	2.11	1.59	0.52	
ρ Per	M4	5.12	1.01	2750	2.30	1.45	0.85	
51 Gem	M4	6.93	2.67	2700	2.36	1.71	0.65	0.70(3)
13 Sge	M4	7.26	3.12	2740	2.31	1.71	0.60	
56 Leo	M5	7.18	2.17	2500	2.65	1.22	1.43	
α Her A	M5	4.99	-0.47	2390	2.83	1.57	1.26	1.32(3)
R Lyr	M5	5.55	0.92	2600	2.50	1.22	1.28	
σ Ceti A max.	M6e	4.80	0.14	2590	2.52	1.17	1.35	
45 Ari	M6	7.05	2.02	2495	2.66	1.07	1.59	1.47(2)

3. The computed Densities.

Table II contains the results of the computations of densities. If separate components are not mentioned, the computed density refers to the brighter component. The subsequent columns give: (1) the β . *G. C.* (or *I. R. C.*) number; (2) the name of the double star; (3) the Mt. W. spectrum when available (the Harvard spectra are in parentheses); (4) the visual magnitudes⁸; (5) the assumed absolute magnitudes⁸; (6) the differences in brightness of the components⁸; (7) the mass ratio, k (directly determined values are printed in italics⁹; the remaining values are according to Eddington's mass-luminosity relation); (8)

⁸ The values for M are taken from⁵ when available; the corrected mean values for m and Δm , given by Öpik in *Publ. Tartu Obs.*, 25.6, 1924, are used; according to Kuiper, for some close pairs the corrections used by Öpik are insufficient (undercorrected) for large differences of magnitude; this circumstance, however, has little influence upon the present statistical material. As to absolute magnitudes, when not in⁵, they were estimated from dynamical parallaxes.

⁹ From R. Huffer's list in *Ap. J.*, 80, 269, 1934.

Table II.

The Densities. An asterisk refers to the Remarks.

Nº	Star	Sp.	<i>m</i>	<i>M</i>	Δm	<i>k</i>	<i>a</i>	<i>P</i>	<i>C</i>	$\log \rho$	ρ
12755	Σ 3062 A	G3	6.5	4.5	0.8	.85	1''.44	106	0.50	0.01	1.02
"	" B	G8	7.3	5.3					0.61	0.19	1.55
21	Σ 2 (A3)	F7	6.8	2.0	0.3	.93	0 .715	275	0.05	-0.77	0.17
104	O Σ 4	F7	8.2	3.4	0.7	.85	0 .41	108	0.39	-0.41	0.39
0h 27	β_2 Tuc	(A2)	4.9	1.9	1.0	.74	0 .48	41.3	-0.03	-0.60	0.25
314	13 Ceti	F7	5.6	4.2	0.7	.85	0 .247	6.89	0.36	-0.19	0.65
335	β 395	G7	6.3	5.4	0.3	.93	0 .66	25.0	0.56	0.03	1.07
374	O Σ 18	F6	7.7	3.4	2.1	.66	0 .96	183	0.19	0.32	2.10
426	η Cas A	F9	3.7	4.8	3.7	.32	12 .21	508	0.41	0.08	1.19
"	B	M0	7.4	8.0					1.34	0.02	1.05*
440	β 232 (F5)		8.7	4.6	0.2	.95	0 .368	91.2	0.28	0.06	1.15
479	O Σ 20 (A0)		6.1	2.0	0.6	.79	0 .57	163	-0.12	-0.72	0.19
482	Σ 73 K1		6.1	2.3	0.5	.88	1 .01	115	0.98	-1.57	0.027
825	β 1000 (G5)		7.2	6.7	4.4	.43	0 .173	4.56	0.55	0.45	2.82
1h 37	p Eri A (G5)		6.0	7.6	0.0	1.00	8 .025	219	0.37	1.53	33.7
"	B (G5)		6.0	7.6					0.37	1.53	33.7
1015	Σ 186 F9		6.8	4.4	0.2	.96	1 .15	136	0.37	-0.10	0.80
1036	β 513 (A3)		4.8	1.5	2.1	1.00	0 .66	63.3	0.11	-0.92	0.12
1070	γ And B (A0)		5.4	1.6	1.4	.70	0 .346	55.0	-0.11	-0.82	0.15
"	C (A2)		6.8	2.8					-0.03	-0.29	0.51
1144	Σ 228 A F2		6.4	3.3	0.8	.83	0 .917	150	0.26	-0.51	0.31
"	B (F5)		7.2	4.1					0.28	-0.13	0.74
1471	β 524 (F0)		5.8	1.2	0.6	.85	0 .16	33.3	0.33	-1.96	0.011
1761	Σ 412 (A2)		6.6	1.0	0.2	.95	0 .49	270	-0.03	-1.24	0.057
2007	Σ 483 A G5		7.4	4.5	1.8	.69	1 .77	135	0.57	0.55	3.56
"	B K1		9.2	6.3					0.71	1.23	17.0
2093	O Σ 77 G0		8.1	3.9	0.1	.97	0 .472	123	0.46	-0.54	0.29
2109	σ^2 Eri B B9		9.7	11.1	1.3	.45	6 .894	248	-0.17	4.51	32200*
"	C M5		11.0	12.7					1.45	-0.52	0.30*
2134	O Σ 79 F6		7.1	3.8	1.8	.68	0 .57	88.9	0.40	-0.44	0.36
2154	O Σ 82 A F8		7.6	3.7	1.1	.78	1 .72	487	0.41	-0.23	0.59
"	B G1		8.7	4.4					0.46	0.24	1.72
2159	β 774 F2		6.3	3.2	0.6	.84	0 .521	64.9	0.26	-0.57	0.27
2187	β 1185 G5		8.2	4.8	0.6	.87	0 .25	28.9	0.36	0.13	1.36
2230	Σ 554 (F0)		5.8	2.2	2.3	.45	1 .036	148	0.08	-0.28	0.52
2381	β 883 F7		7.7	3.7	0.0	1.00	0 .19	16.6	0.30	0.03	1.08
2383	β 552 F5		6.8	3.3	3.3	.48	0 .56	86.0	0.12	-0.09	0.81
2535	O Σ 98 A F2		5.9	2.6	0.6	.83	1 .22	190	0.19	-0.52	0.30
"	B (A2)		6.5	3.2					-0.03	0.14	1.38
2597	Capella A G1		0.7	0.1	0.4	.79	0 .054	0.29	0.75	-3.00	0.0010
"	B F5		1.1	0.5					0.39	-2.25	0.0056
3291	β 895 (A3)		7.8	3.3	0.0	1.00	0 .255	45.7	0.05	0.03	1.06
3474	O Σ 149 G2		7.0	4.2	2.5	.58	0 .77	103	0.50	-0.40	0.40
3596	Sirius A (A0)		—1.6	1.2	10.0	.47	7 .57	50.0	0.05	-1.14	0.072
"	B F0		8.4	11.2					0.16	4.34	21800*
"	I 65 (F5)		6.8	4.8	0.2	.96	0 .315	16.5	0.28	0.20	1.58
3876	Σ 1037 A F6		7.1	3.7	0.2	.97	0 .87	120	0.33	-0.11	0.78
"	B F5		7.3	3.9					0.30	0.05	1.12
4122	Castor A (A0)		2.0	1.6	0.8	.79	6 .57	478	-0.08	-1.00	0.10*
"	B (A7)		2.8	2.4					0.10	-0.92	0.12
4187	Procyon F3		0.5	3.1	12.0	.35	4 .26	40.2	0.28	-0.80	0.16

Table II. Continued.

Nº	Star	Sp.	<i>m</i>	<i>M</i>	Δm	<i>k</i>	<i>a</i>	<i>P</i>	<i>C</i>	$\log \varrho$	ϱ
4310	β 101	G2	5.8	4.8	0.6	.88	0''.69	23.3	0.46	0.03	1.07
4355	O Σ 185	F6	7.1	3.7	0.3	.93	0 .35	59.6	0.23	-0.51	0.31
4414	β 581	K2	8.7	5.6	0.0	1.00	0 .38	44.0	0.82	-0.20	0.63
4477	ζ Can A	F7	5.6	3.6	0.6	.85	0 .874	57.9	0.52	-0.67	0.21
4771	ε Hya	F8	3.8	0.7	1.5	.35	0 .23	15.3	0.71	-2.52	0.0030
4923	σ_2 U Ma	F4	4.9	3.5	3.3	.49	4 .76	470	0.35	-0.31	0.49
5005	Σ 3121	K4	7.9	6.3	0.3	.93	0 .669	34.0	0.96	0.06	1.14
5103	ω Leo	F8	5.9	3.8	0.9	.81	0 .844	117	0.49	-1.08	0.083
9h 40	ψ Arg	(F5)	3.6	2.3	2.0	.62	0 .914	34.9	0.30	-0.89	0.13
5223	φ U Ma	(A2)	5.2	1.3	0.2	.93	0 .343	113	-0.09	-1.68	0.021
5235	A. C. 5	(A2)	5.7	1.2	0.4	.87	0 .41	72.8	-0.03	-0.85	0.14
5734	ξ U Ma A	G0	4.4	4.3	0.5	.47	2 .52	59.8	0.44	0.20	1.58
	B	G0	4.9	4.6					0.42	0.21	1.61
5765	Σ 1536 A	F4	4.1	2.6	2.7	.54	1 .92	183	0.22	-0.96	0.11
	B	F5	6.8	3.1					0.30	0.27	1.85
5805	O Σ 234	F1	8.1	3.2	0.4	.89	0 .347	84.7	0.25	-0.24	0.57
5811	O Σ 235	F4	5.7	3.5	1.4	.72	0 .78	71.9	0.29	-0.52	0.30
5951	β 794	F7	7.1	3.5	1.3	.75	0 .34	63.1	0.39	-0.82	0.15
6028	Σ 3123	F5	7.8	3.0	0.2	.95	0 .22	103	0.31	-0.82	0.15
6158	Σ 1639	A5	6.6	1.9	1.2	.69	1 .00	361	0.06	-0.66	0.22
12h 61	γ Cen A	A0	3.1	0.4	0.0	1.00	1 .924	203	-0.26	-0.92	0.12
	B	A0	3.1	0.4					-0.26	-0.92	0.12
6243	γ Vir A	F0	3.7	2.8	0.0	1.00	3 .62	178	0.22	-0.46	0.35
	B	F0	3.7	2.9					0.22	-0.46	0.35
6406	Σ 1728 A	F4	5.2	3.6	0.1	.98	0 .665	25.9	0.15	0.04	1.10
	B	F5	5.3	3.7					0.17	0.09	1.23
6524	O Σ 269 A	A5	7.3	2.1	0.5	.88	0 .325	48.8	0.06	0.00	0.99
	B	A7	7.8	2.6					0.10	0.17	1.47
6530	Σ 1757	K1	7.7	6.0	1.1	.78	1 .78	244	0.73	-0.07	0.85
6566	Σ 1768	(F0)	5.1	2.2	2.0	.58	1 .205	220	0.04	-0.82	0.15
6578	β 612	(F2)	6.3	1.5	0.1	.96	0 .225	23.0	0.21	-0.70	0.20
6641	Σ 1785 A	K6	7.9	7.0	0.4	.91	2 .47	151	1.13	0.18	1.52
	B	K6	8.3	7.4					1.10	0.43	2.68
6711	β 1270 A	F5	8.5	3.0	0.1	.97	0 .21	38.1	0.31	-0.09	0.81
	B	F5	8.6	3.1					0.30	-0.03	0.94
6780	Σ 1819	F7	7.8	4.1	0.1	.97	1 .11	142	0.37	0.41	2.58
6832	Σ 1834	(F8)	8.0	3.9	0.1	.97	0 .93	296	0.32	-0.25	0.56
6842	β 1111 B	F2	7.3	3.0	0.1	.98	0 .235	40.5	0.26	-0.64	0.23
6913	A 570	(A2)	6.6	2.0	0.2	.94	0 .202	28.4	-0.03	-0.43	0.37
14h 59	α Cen A	G4	0.3	4.8	1.4	.85	17 .66	80.1	0.44	-0.06	0.87
	B	K5	1.7	6.2					1.05	-0.35	0.45
6955	ζ Boo A	(A2)	4.5	2.1	0.3	.96	0 .62	130	-0.08	-1.48	0.033
	B	(A2)	4.8	2.4					-0.08	-1.31	0.049
6999	Σ 1879	G1	7.6	4.5	1.0	.80	0 .789	178	0.49	-0.51	0.31
7001	O Σ 285	F5	7.7	3.2	0.5	.88	0 .33	88.5	0.30	-0.68	0.21
7034	ξ Boo A	G5	4.8	5.3	2.0	.96	4 .87	151	0.62	0.06	1.15
	B	K5	6.8	6.9					1.01	0.58	3.78
7120	Σ 1909 A	G1	5.3	4.5	0.7	.59	3 .58	205	0.50	-0.01	0.98
	B	G2	6.0	5.2					0.44	0.29	1.93
7251	Σ 1937	F9	5.6	4.1	0.5	.88	0 .89	41.6	0.38	-0.12	0.75
7259	μ Boo B	G0	7.2	4.5	0.5	.88	1 .30	224	0.51	-0.36	0.44
15h 55	γ Lup A	(B3)	3.6	-0.5	0.2	.92	0 .78	104	-0.27	-1.19	0.065
	B	(B3)	3.8	-0.3					-0.27	-1.11	0.078

Table II. Continued.

Nº	Star	Sp.	<i>m</i>	<i>M</i>	<i>A m</i>	<i>k</i>	<i>a</i>	<i>P</i>	<i>C</i>	<i>log ρ</i>	<i>ρ</i>
7322	$O\Sigma$ 298	K4	7.4	6.3	0.3	.93	0''.88	56.7	0.96	-0.34	0.46
7368	γ Cor Bor	(A0)	4.0	0.4	2.5	2.03	0 .62	101	-0.19	-1.55	0.028
7416	π_2 U Mi	F0	7.3	2.5	1.0	.78	0 .42	115	0.12	-0.49	0.32
7487	ξ Sco A	F4	4.8	3.0	0.2	.94	0 .72	44.7	0.29	-0.66	0.22
"	B	F5	5.0	3.2					0.30	-0.66	0.22*
7563	σ Cor Bor A	F6	5.8	3.7	0.9	.92	6 .00	900	0.32	-0.10	0.80
"	B	G1	6.7	4.0					0.48	0.17	1.48
7561	Σ 2026	K6	9.4	7.2	0.5	.88	1 .53	215	1.12	0.17	1.48
7642	Σ 2052	K2	7.7	5.7	0.0	1.00	2 .87	318	0.79	0.17	1.47
7649	λ Oph	(A0)	4.1	1.0	1.4	1.00	0 .94	135	-0.08	-1.21	0.061
7717	ζ Her	G0	3.1	3.7	3.1	1.04	1 .35	34.5	0.48	-1.12	0.075
7748	D 15	K6	8.9	7.0	0.2	.96	0 .935	126	1.12	-0.33	0.47
7783	Σ 2107	F5	6.7	3.5	1.5	.69	0 .783	154	0.30	-0.59	0.26
13364	Hu 1176	(A5)	6.1	2.1	0.0	1.00	0 .16	15.5	0.06	-0.68	0.21
17h 31	Lac 7194	(K0)	5.6	6.5	2.8	.58	3 .50	101	0.70	0.42	2.62
7929	β 416	K5	5.9	7.0	2.0	.75	1 .83	42.2	1.00	-0.05	0.90
7936	$O\Sigma$ 327	F4	8.5	3.5	0.3	.93	0 .26	88.0	0.29	-0.49	0.32
8038	Σ 2173	G6	6.0	4.8	0.2	.97	1 .06	46.0	0.57	-0.08	0.83
8099	β 962	G1	5.3	4.2	4.5	.89	1 .51	80.6	0.47	-0.38	0.42
8162	μ Her B	M3	10.2	10.2	0.6	1.00	1 .30	43.2	1.34	0.31	2.06*
8303	τ Oph A	F3	5.3	2.5	0.7	.83	1 .307	224	0.27	-1.07	0.085
"	B	F3	6.0	3.2					0.27	-0.72	0.19
17h 129	h 5014 A	(A3)	5.8	2.6	0.0	1.00	1 .146	180	0.05	-0.43	0.37
"	B	(A3)	5.8	2.6					0.05	-0.43	0.37
8340	70 Oph A	K1	4.3	5.9	1.7	1.00	4 .56	89.1	0.79	-0.17	0.68
"	B	K6	6.0	7.3					1.11	0.31	2.04
8353	$O\Sigma$ 341	G1	7.9	3.6	0.9	.81	0 .30	19.8	0.51	0.28	1.89
8372	99 Her	F5	5.2	4.1	3.9	.43	1 .03	56.0	0.29	-0.16	0.69
8380	Σ 2281	(F2)	6.0	1.7	1.2	.71	1 .33	424	0.24	-1.09	0.081
8679	A 88 A	F8	7.1	3.9	0.3	.93	0 .176	12.1	0.40	-0.31	0.49
"	B	F8	7.4	4.2					0.39	-0.15	0.71
8933	β 648	G0	5.3	4.6	3.0	.54	1 .24	61.8	0.37	-0.11	0.77
8966	Σ 2438	(A2)	6.7	1.3	0.6	.81	0 .53	233	-0.03	-0.92	0.12
8965	ζ Sgr	(A2)	3.3	1.5	0.4	.87	0 .565	21.2	-0.01	-0.85	0.14
18h 113	γ Cor Au A	F7	5.0	3.6	0.0	1.00	2 .14	125	0.38	-0.33	0.47
"	B	F7	5.0	3.5					0.38	-0.33	0.47
9114	Se 2 B	K0	8.7	5.5	0.0	1.00	0 .40	58	0.68	-0.14	0.73
9319	Σ 2525	F9	8.5	3.8	0.2	.95	1 .205	355	0.44	0.03	1.07
9605	δ Cyg	(A0)	3.0	0.5	4.4	.35	2 .12	321	-0.21	-1.17	0.067
9643	ζ Sge	(A2)	5.5	2.2	0.6	.83	0 .32	25.2	-0.12	-0.21	0.61
9650	$O\Sigma$ 387	F2	7.1	3.4	0.4	.91	0 .566	128	0.26	-0.58	0.26
9979	$O\Sigma$ 400	G4	7.5	4.6	1.0	.80	0 .428	84.4	0.54	-0.82	0.15
10363	β Del	F3	4.0	2.6	1.3	.69	0 .48	26.8	0.31	-1.35	0.045
10559	Σ 2729	F3	6.4	3.1	0.8	.81	0 .695	152	0.27	-0.90	0.13
10643	ε Equ	F0	6.0	2.9	0.5	.86	0 .61	97.4	0.35	-1.06	0.087
10732	61 Cyg A	K6	5.6	7.7	0.7	2.22	32 .8	756	1.16	0.49	3.08
"	B	M0	6.3	8.6					1.55	0.48	2.99*
10829	δ Equ	F3	5.2	4.0	0.5	.88	0 .27	5.70	0.42	-0.26	0.55
10846	τ Cyg	F0	3.9	2.7	3.2	.67	0 .96	49.2	0.29	-1.00	0.10
11125	24 Aqr	F7	7.3	3.8	0.8	.83	0 .41	46.6	0.37	-0.19	0.65
11222	α Peg	F2	4.8	3.0	0.4	.56	0 .22	11.5	0.23	-0.96	0.11
11761	Kr 60 A	M3	9.3	10.8	1.5	.59	2 .36	44.5	1.34	0.63	4.22*
"	" B	M4	10.8	12.3					1.45	0.40	2.50*

Table II. Continued.

Nº	Star	Sp.	<i>m</i>	<i>M</i>	Δm	<i>k</i>	<i>a</i>	<i>P</i>	<i>C</i>	$\log \varrho$	ϱ
11763	Σ 2912	F2	5.9	2.3	0.7	.82	<i>0''.72</i>	136	<i>0.16</i>	-0.85	0.14
12143	A 417	(F0)	6.3	2.8	0.0	1.00	0 .245	23.8	0.16	-0.55	0.28
12196	π Ceph	G1	4.6	0.5	3.1	.39	0 .92	178	<i>0.72</i>	-2.40	0.0040
12290	β 80	K1	8.6	5.8	0.1	.98	0 .79	85.7	<i>0.74</i>	0.26	1.80
12304	Σ 3001	G7	5.0	0.7	2.5	.48	2 .81	177	<i>0.85</i>	-0.96	0.11
12404	β 1266	F5	8.4	3.3	0.1	.97	0 .22	40.0	0.30	-0.11	0.77
12419	Hu 298	(F5)	7.5	3.8	0.2	.96	0 .242	30.0	<i>0.32</i>	-0.31	0.49
12696	Hdn 60	K1	9.2	5.9	0.4	.91	0 .50	40.8	<i>0.73</i>	0.70	4.96
12701	β 733	G1	5.9	4.7	4.0	.82	0 .82	26.3	0.59	0.00	1.01

the semi-major axis¹⁰; (9) the period¹⁰; (10) the colour index; observed values⁴ are in italics (for № 7034 and 8340 the observed colour indices refer to the combined light of both components; the colours of the separate components were computed with the aid of Table 11 in⁴); (11) the logarithm of the density; (12) the density (sun = 1).

Remarks.

- 426 (η Cas B). — See table III.
- 2109 (σ^2 Eri B). — Spectrum by Leonard¹¹.
- 2109 (σ^2 Eri C). — See table III.
- 3596 (Sirius B). — With Vyssotsky's mean value $m = 7.0$ the density would be 3150.
- 4122 (Castor) — Since both components of Castor are spectroscopic binaries, their real surfaces are about one half of the adopted surfaces (assuming equality of both spectroscopic components). Their volumes are $2^{\frac{3}{2}}$ times smaller, their masses one-half of the dynamical mass; therefore the computed densities must be multiplied by $\frac{1 \cdot 2^{\frac{3}{2}}}{2}$, or by $\sqrt{2}$, to get the true densities. We have for each spectroscopic component of A, $\varrho = 0.14$; and for the components of B, $\varrho = 0.17$.
- 7487 (ξ Sco B) — Spectrum by Leonard¹¹.
- 8162 (μ Her B). — See table III.
- 10732 (61 Cyg B). — See table I.
- 11761 (Kr 60 A). — See table III.
- 11761 (Kr 60 B). — Spectrum by Leonard¹². See table III.

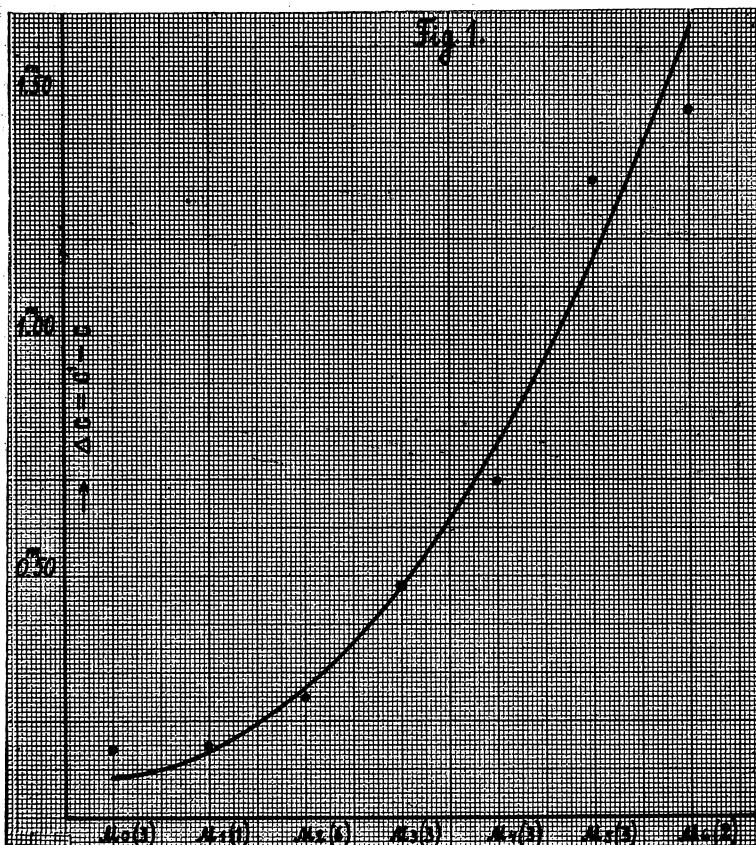
¹⁰ The data in columns (8) and (9) are taken from van den Bos' "Table of Orbits", in *B. A. N.*, **3**, 149, 1927, and supplemented by a few more recent determinations.

¹¹ Fr. C. Leonard, *Lick Obs. Bull.*, **10**, 169, 1923.

¹² Fr. C. Leonard, *Publ. A. S. P.*, **39**, 362, 1927.

4. The Densities of Red Dwarfs.

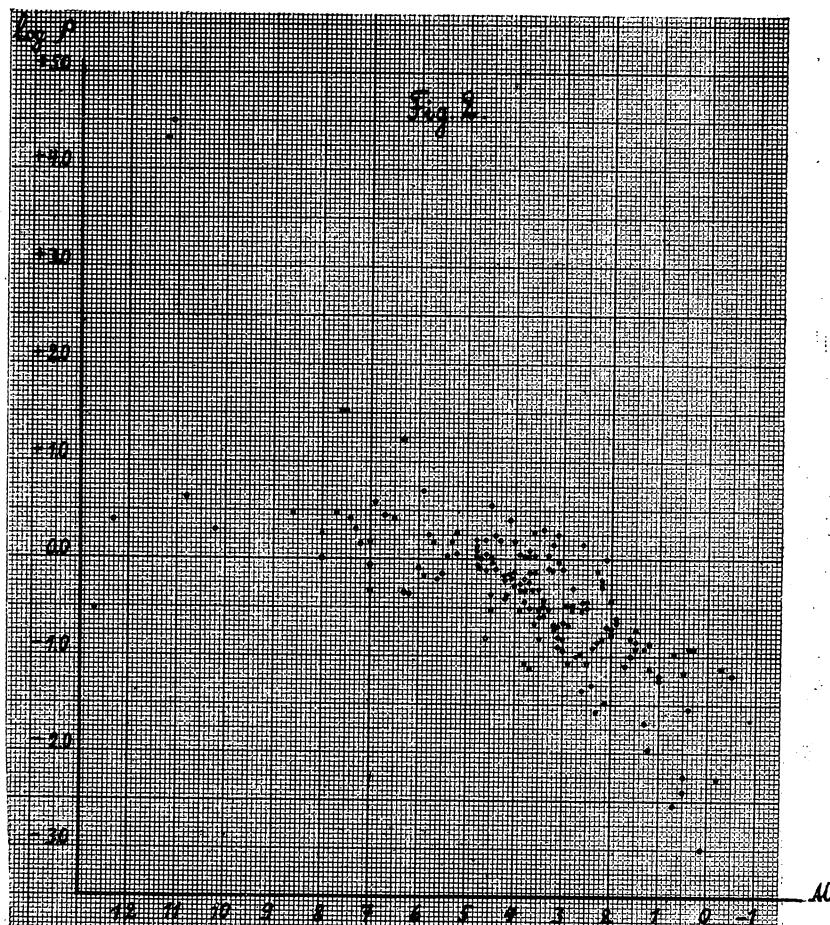
An unexpected result is obtained by computing the densities of the class M dwarf stars. A wide-spread opinion among astronomers is that the M dwarfs must have the greatest densities along the normal dwarf branch. Our computations show



The observed $Ti\ O$ correction of colour as function of spectral type. Abscissae, spectral type, with numbers of individual stars in parentheses; ordinates, the $Ti\ O$ correction of colour index.

that this opinion is not well founded. The $Ti\ O$ correction brings the densities of the red dwarfs from the abnormally high values down to the "normal" densities of other dwarfs. Thus these stars cannot be counted with the dense, or "white" dwarfs any more. On the contrary, their densities come out on the average even smaller than those of some other groups of dwarf stars, as shown by Fig. 2, and by Table IV below. Table II contains a comparison of the corrected values (ρ') with the uncorrected densities (ρ). Columns (6), (7) and (8) give the $Ti\ O$ correction (from Fig. 1), the corrected colour index, and the corrected "ideal" apparent magnitude respectively. The differ-

ence between ϱ and ϱ' increases rapidly with the advancing M spectrum, in agreement with the rapid growth of the TiO effect.



Density and absolute magnitude. Abscissae, absolute magnitudes; ordinates, logarithms of density.

Table III.
Densities of M Dwarfs corrected for TiO Effect (ϱ'),
as Compared with the Uncorrected Densities (ϱ).

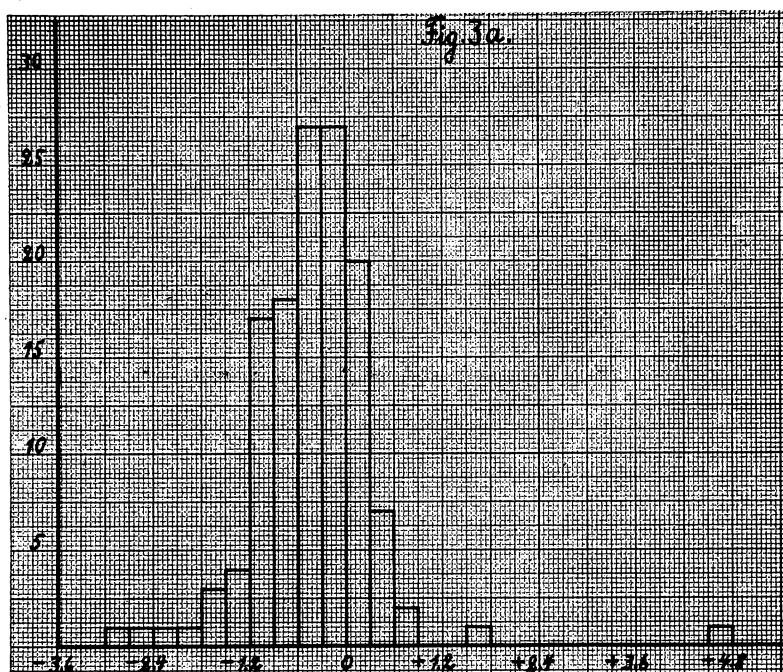
Nº	Star	Sp.	m	C	ΔC	C'	m'	ϱ	ϱ'
426	η Cas B	M0	7.4	1.34	0 ^m .08	1.42	7.32	1.64	1.05
8162	μ Her B	M3	10.2	1.34	0 .48	1.82	9.72	26.5	2.06
11761	Kr. 60 A	M3	9.3	1.34	0 .48	1.82	8.82	54.5	4.22
11761	Kr. 60 B	M4	10.8	1.45	0 .79	2.24	10.01	165	2.50
2109	σ^2 Eri C	M5	11.0	1.45	1 .17	2.62	9.83	150	0.30

It must be remarked that the knowledge of the exact spectral subdivision of a red dwarf is of very great importance

for our purposes, on account of the rapid change of ΔC with spectrum. Therefore a slight error in the spectrum produces a much greater error in the assumed "true" colour index, consequently in the density also.

5. Statistical Discussion and Mean Densities.

Fig. 2 represents the correlation of luminosity and density. Up to the value $M=5$ a general increase of density with



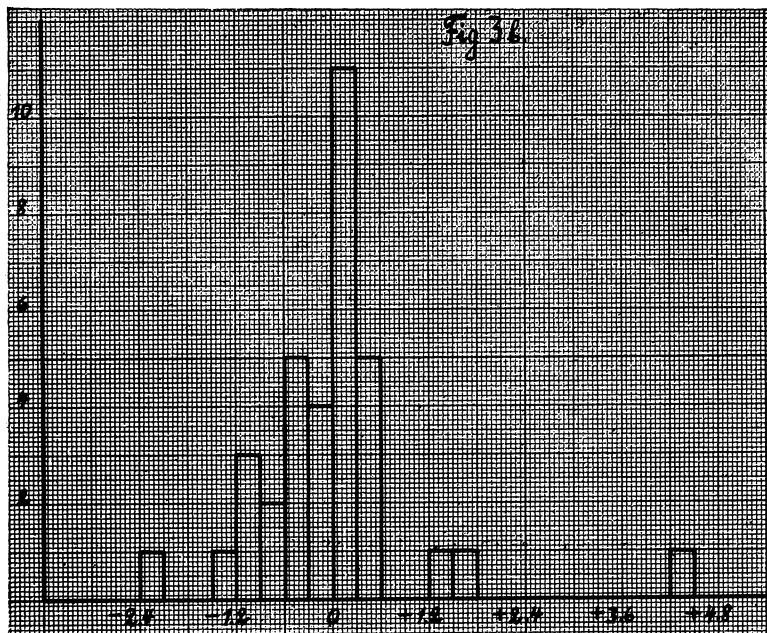
Frequency of density logarithms of the brighter components. Abscissae, $\log \varphi$; ordinates, observed frequency.

decreasing luminosity is noticeable. For stars of lower luminosity the luminosity — density curve seems to be split into two branches, one leading to the white dwarfs, the other to the red ones. The first six dots from the left of the figure are: above, the white dwarfs Sirius *B* and σ^2 Eridani *B*; below, the *M* dwarfs Krueger 60 *A* and *B*, μ Herculis *B* and σ^2 Eridani *C*.

Fig. 3a and 3b represent the frequency of density logarithms: 3a, of the brighter components, and 3b, of the fainter ones. The densities of the latter are, on the average, greater than of the principal components, a circumstance which agrees well with the general trend of Fig. 2.

Fig. 3a may also be regarded as representing to some

extent the distribution of densities for stars as a whole, because the principal components of the binaries probably may be regarded as a fair statistical sample of our stellar system (observational selection, of course, playing a certain rôle). We notice that the distribution of $\log \rho$ is approximately symmetrical, a circumstance which has been revealed by Öpik's much less



Frequency of density logarithms of the fainter components. Abcissae, $\log \rho$; ordinates, observed frequency.

complete material¹. As to the question of reality of the gap between normal and white dwarfs, revealed by Fig. 3a and 3b, it can be decided only on the basis of a material at least twenty times more abundant than ours.

Table IV gives the relation between spectrum and density for the normal dwarf stars. In agreement with the character of the density distribution as revealed by fig. 3a, b, logarithmic mean densities were computed. The fourth column of the table gives the probable error of the mean density logarithm, computed from $\pm \frac{\Delta_0}{\sqrt{n}}$, where Δ_0 denotes the probable deviation of the density logarithm from its mean of the given spectral class.

From all spectra together we found $\Delta_0 = \pm 0.674 \sqrt{\frac{\sum \Delta^2}{(\Sigma n) - k}} = \pm 0.286$, where k is the number of separate groups, Δ — the individual deviation of $\log \rho$ from the mean of the group. The

maximum observational error in $\log \varrho$ as computed from (8), (8a), may be estimated as $\Delta_0 = \pm \sqrt{\Delta_a^2 + \Delta_k^2 + (0.6 \Delta_m)^2 + (1.71 \Delta_c)^2}$, where Δ_a is the p. e. of $\log \frac{a^3}{P^2}$, Δ_k — the p. e. of $\log (1+k)$, Δ_m — the photometric error in visual magnitude, Δ_c — the p. e. of colour. We assume: $\Delta_a = \pm 0.129$ (corresponding to p. e. ± 10 per cent in a); $\Delta_m = \pm 0.2$; $\Delta_c = \pm 0.05$; $\Delta_k = \pm 0.02$. This gives us $\Delta_0 = \pm 0.197$ and the cosmic spread in $\log \varrho$ for a given spectral group and for normal dwarfs $\Delta = \pm \sqrt{0.286^2 - 0.197^2} = \pm 0.207$, or a probable deviation in the ratio of 1,61:1.

Table IV.
Relation of Spectrum and Mean Density for the
Normal Dwarf Branch.

Sp.	$\log \varrho$	ϱ	$\pm \frac{\Delta_0}{\sqrt{n}}$	n
B3	-1.15	0.071	± 0.20	2
A0—A2	-0.91	0.12	± 0.06	21
A3—A7	-0.46	0.35	± 0.09	10
F0—F3	-0.70	0.19	± 0.06	24
F4—F6	-0.24	0.57	± 0.05	28
F7—G1	-0.21	0.61	± 0.05	31
G2—G6	+0.25	1.77	± 0.08	13
G7—K1	+0.27	1.87	± 0.10	9
K2—K6	+0.09	1.23	± 0.08	13
M0—M5	+0.22	1.66	± 0.12	6

The data in Table IV indicate a more or less steady increase of the density with advancing spectral type of the normal dwarf branch until the class K is reached; remarkable is here, however, the relapse of density for F0—F3, perhaps due to an insufficient separation of giants and dwarfs in this group. From K dwarfs on, the mean density appears to be constant, within the limits of accidental statistical fluctuations. In conclusion, it may be noticed that an attempt to find, on the basis of our material, any relationship between the density and period, or eccentricity of a double star gave no positive result.

6. The Mass-Luminosity Relation.

The red dwarfs, used by Eddington¹³ in his mass-luminosity diagram showed a slight systematic deviation from the

¹³ A. S. Eddington, *M. N.*, 84, 308, 1924.

theoretical curve. The absolute bolometric magnitude of these stars is evidently underestimated by the theoretical curve. At present we may obtain more accurate estimates of the theoretical, and the true absolute bolometric magnitudes of these stars, using the photographic determination of mass, given by Huffer⁹, and the TiO correction, derived in this paper. Table V contains the results of this computation.

Table V.
Comparison of Observed and Theoretical Absolute Magnitudes of Red Dwarfs.

Star	Sp.	T	mass	$M_{\text{vis.}}$	M_{Bol}	M_{calc}	$O-C$
μ Her B	M3	3180	0.44	9.51	8.03	8.78	-0.75
Kr 60 A	M3	3180	0.25	10.75	9.27	11.41	-2.14
Kr 60 B	M4	2800	0.14	11.94	9.85	14.29	-4.44
σ^2 Eri C	M5	2520	0.20	10.30	7.70	12.67	-4.97

The temperature was computed from formula (7), by using the corrected colour indices, given in Table III. For the absolute visual magnitude we used the same values of the trigonometric parallaxes as used by Eddington¹³, and the corrected apparent magnitudes from Table III. The column M_{Bol} gives the absolute bolometric magnitude derived by applying Eddington's correction¹⁴ to the corrected absolute visual magnitude. M_{calc} is deduced from the mass, according to Table I in¹³, the temperature term $-2 \log \frac{T}{5200}$ being added. The last column contains the residuals (observed — calculated). As follows from the data, the systematic deviation from the theoretical curve is for the red dwarfs much greater than one could expect, making almost linear the empirical relation between \log mass and absolute magnitude.

Tartu, May, 1935.

¹⁴ A. S. Eddington, *M. N.*, 77, 605, 1917.