PUBLICATIONS
DE
L'OBSERVATOIRE ASTRONOMIQUE DE L'UNIVERSITÉ DE TARTU
(DORPAT)
TOME XXV № 5

ON THE LUMINOSITY-CURVE OF COMPONENTS OF DOUBLE STARS

BY

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TARTU 1923

Abbreviation.

This series of publications is denoted for brevity's sake by the initial letters *T.P.*

K. Mattiesen, Tartu (Dorpat).

On the Luminosity-Curve of Components of Double Stars.

By E. Öpik.

1. Introduction.

The frequency-function of stellar luminosities, or the so-called "Luminosity-Curve", plays an important part in the study of the structure of the universe. Recently we also attempted an explanation of the Luminosity-Curve from the standpoint of stellar evolution[1]); on the basis of certain hypotheses, the chief of which consists in the assumption of a reiteration of the stages of stellar evolution, a distribution of luminosities can be derived which is similar to the observed Lnminosity-Curve. The concrete assumptions on which the numerical computation was based were of necessity somewhat arbitrary, and therefore a mere coïncidence of the theory with observed data cannot be regarded as a definitive test in favour of the former; the foundation of the different hypotheses which formed the basis of the theory must be sought in independent observational data.

One of the best opportunities to penetrate into the laws of stellar evolution presents the study of double stars. In the case of a close system forming a physical pair we can be sure that the time elapsed since the stars began their evolution as separate bodies is equal for both components; therefore, if certain differences in their physical properties appear, they must be ascribed to different conditions of evolution, e. g. the difference in the mass, in the inner structure, etc; it may be hoped in this way to obtain information on the influence of various factors upon the rate and direction of the evolution.

In dealing with double or multiple stars it must be remembered, however, that lines of analogy between the components

1) *Theoretical Luminosity-Curves and Stellar Evolution.* *T.P.* 25_2 (1922).

of these systems and the single stars must be drawn with great caution; the mere fact that in one case the matter condensed into two or more nearly equivalent centra, in the other — that only one centre was formed, indicates that different factors acted upon them. A mutual influence of the components upon one another can be *a priori* expected in the distribution of the masses and luminosities; and from this standpoint it may be questioned, e. g., whether it is legitimate to count components of known double stars together with ordinary single stars [1]) in such problems as the derivation of the Luminosity-Curve of stars in the immediate neighbourhood of our sun, — a method of counting used by several authors.

These and similar considerations led to the present discussion; from the observational data available it seemed possible to throw some light on the question — what is the distribution of luminosities for components of double stars?

2. Derivation of the Distribution for Close Pairs.

Let M and M' be the absolute magnitudes of the components; and let $M'-M=r$, so that r denotes the difference of their magnidudes; let $\psi(M)$, $\varphi(M')$ and $\chi(M',r)$ be the frequency-functions of M, M' and r respectively; then the following equation may be written:

$$\varphi(M') = \int_a^\infty \psi(M'-r) \cdot \chi(M',r)\, dr \quad . \; . \; . \qquad (1).$$

The inferior limit, a, is zero when M' denotes the magnitude of the secondary component, so that $M' \geqslant M$; however, when the cases $M' > M$ and $M' < M$ can equally occur — as in the third components of certain pairs — $a = -\infty$.

Two extreme cases may be considered:

1) when $\frac{\partial\chi}{\partial M'} = 0$, i. e. when the frequency-function of the difference r depends on this difference only; such a case would correspond to a high degree of dependence of the magnitude of the secondary upon the magnitude of the primary component;

1) Which can be undiscovered binaries.

without knowing the distribution ψ the distribution φ cannot be deduced, but the function χ can be easily determined from every list of observational data, the result being uninfluenced by *selection* in the magnitudes M;

2) when $\frac{\partial \chi}{\partial r} = 0$, so that the observed frequency depends on the magnitude of the secondary component only; this case corresponds to the magnitudes of the components being entirely independent of one another; equation (1) will be transformed then into

$$\varphi(M') = \int_{-\infty}^{+\infty} \psi(M'-r)\,\chi(M')\,dr, \text{ or, with } r = M'-M,$$

$$\varphi(M') = \chi(M') \int_{-\infty}^{+\infty} \psi(M)\,dM = c\chi(M') \quad . \; . \; . \quad (1');$$

thus the true distribution of the magnitudes, $\varphi(M')$, will be identical with the observed distribution, $\chi(M')$, differing from the latter only by the constant factor c; therefore in this case the true frequency-function of the magnitudes of the components can be derived directly from every observational material without fearing the danger of being affected by selection in M.

The material discussed here consists of two lists given by J. Jackson and H. Furner[1]), and by J. Jackson[2]); after excluding from one list the pairs in common for both lists and those with unknown spectra, there remains for the statistical discussion a total of 645 pairs. Although our discussion is thus limited to only a small fraction of all double stars known at present, this may be regarded as a gain, since the material chosen consists generally of very close pairs with small absolute separation of the components, so that considerable homogeneity is attained.

Table 1 contains the number of pairs with given differences of magnitude, arranged according to the spectral type of the brighter component; giants from F to M are joined together; the other spectral subdivisions refer to the dwarf branch of the

1) *Hypothetical Parallaxes of 556 Visual Double Stars.* Monthly Notices, **81**, pp 2—31.

2) *The W. Struve (Σ) Double Stars.* Ibidem, **83**, pp 4--32.

Table 1.

Distribution of Differences in Magnitude of the Components of the Pairs in Motion.

m = magnitude of the brighter component.
m' = „ „ „ fainter component.
m_c = combined magnitude of the pair.

$m_c \leqslant 6.00$; $\overline{m_c} = 5.0$

Sp. Type	Giants	B_0—B_9	A_0—A_5	F_0—F_8	G_0—G_5	K_0—K_5	M	Sum	m'	$1:\pi$
$m - m$ (r)	Number									
0.0—0.4	2	3	15	8	1	0	0	29	5.0	1.00
0.5—0.9	2	5	8	7	7	1	0	30	5.5	1.00
1.0—1.9	5	5	14	13	3	0	0	40	6.5	1.00
2.0—2.9	3	3	10	7	1	2	0	26	7.5	1.00
3.0—3.9	5	1	6	7	2	0	0	21	8.5	1.33
4.0—4.9	0	0	2	3	0	0	0	5	9.5	2.78
$\geqslant 5.0$	3	0	2	4	3	0	0	12	—	—
	(6.0; 7.9; 5.2)		(10.0; 5.5)	(13.0; 5.0; 5.5; 5.8)	(6.5; 5.1; 6.8)					
Total	20	17	57	49	17	3	0	163	—	—

$6.01 \leqslant m_c \leqslant 7.99$; $\overline{m_c} = 7.0$

Sp. Type	Giants	B_0—B_9	A_0—A_5	F_0—F_8	G_0—G_5	K_0—K_5	M	Sum	m'	$1:\pi$
$m' - m$ (r)	Number									
0.0—0.4	7	5	26	55	15	7	0	115	7.0	1.00
0.5—0.9	4	5	19	32	15	3	0	78	7.5	1.00
1.0—1.9	3	6	24	35	15	4	0	87	8.5	1.33
2.0—2.9	0	1	3	17	9	0	0	30	9.5	2.78
3.0—3.9	0	0	5	6	3	2	0	16	10.5	4.76
4.0—4.9	1	0	1	1	1	1	0	5	11.5	11.0
$\geqslant 5.0$	0	0	0	0	0	0	0	0	12.5	40
Total	15	17	78	146	58	17	0	331	—	—

$m_c \geqslant 8.0$; $\overline{m_c} = 8.5$

Sp. Type	Giants	B_0—B_9	A_0—A_5	F_0—F_8	G_0—G_5	K_0—K_5	M	Sum	m'	$1:\pi$
$m' - m$ (r)	Number									
0.0—0.4	1	0	8	18	24	10	0	61	8.5	1.33
0.5—0.9	2	1	4	12	10	7	1	37	9.0	2.10

Table 1. Continued.

$m_c \geqslant 8.0$; $\overline{m}_c = 8.5$

Sp. Type	Giants	B_0—B_9	A_0—A_5	F_0—F_8	G_0—G_5	K_0—K_5	M	Sum	m'	$1:\pi$
$m'-m$ (r)	Number									
1.0—1.9	0	1	8	9	12	5	0	35	10.0	3.70
2.0—2.9	1	0	3	3	3	3	0	13	11.0	6.67
3.0—3.9	0	0	0	0	2	1	1	4	12.0	22
4.0—4.9	0	0	0	1	0	0	0	1	13.0	67
$\geqslant$5.0	0	0	0	0	0	0	0	0	—	—
Total	4	2	23	43	51	26	2	151	—	—
Total All Magn.	39	36	158	238	126	46	2	645	—	—

spectral series; the separation of giants and dwarfs was made on the basis of the absolute magnitudes given in the lists of the hypothetical parallaxes mentioned above; the faintest absolute magnitude which occurred among the stars classified as Giants was: M, 0.9; K_2, 1.1; K_0, 3.2; G_8, 2.0; G_5, 2.5; G_0, 2.2; F_8, 0.8 ($\pi = 0''.1$); it must be taken into account that these hypothetical magnitudes as given by Jackson and Furner are somewhat too low if compared with the Mt. Wilson results. In other respects the hypothetical magnitudes were not used.

The difference of magnitudes, $m'-m$, was adopted according to Burnham's General Catalogue of Double Stars; the scale of these magnitudes, based on simple eye-estimates, is somewhat uncertain; for a small number of pairs photometric magnitudes determined at Harvard are available, but these magnitudes were not used because they would introduce only a certain non-homogeneity without altering the general character of the material. The data of the table are divided into 3 groups: 1) pairs brighter than the 6th magnitude, with the average magnitude adopted $= 5.0$; 2) pairs with combined magnitude from 6.01 to 7.99, average magnitude 7.0; 3) pairs fainter than 8.0, average magnitude adopted $= 8.5$.

The numbers of table 1 are strongly influenced by the selection in the apparent magnitudes and the differences $m'-m$; the discovery of faint components of close pairs is more difficult than the discovery of bright ones, and therefore the latter must be represented more abundantly in the lists than the former; a selection depending on the combined apparent magnitude must also take place, the fainter pairs being on the average more distant and, consequently, of a smaller angular separation than the brighter pairs; the result of this selection is the gradual increase of the percentage of late-type dwarf systems as the apparent brightness diminishes; in other respects, however, the said selection is of no importance, since only relative numbers for different values of $m'-m$ are needed here, and for a given combined apparent magnitude all numbers are changed by the selection in the same ratio.

Let p denote the percentage of discovered objects, or the "coefficient of perception" for a given category of double stars. We make the assumption that p depends on two arguments, the apparent magnitude of the fainter component, m', and the difference of magnitudes, $r = m' - m$, and that the influence of both arguments can be represented by the product of two independent factors as follows:

$$p = \pi(m') \cdot \lambda(r) \quad . \; . \; . \quad (2),$$

where π and λ are certain functions, both *decreasing* with increasing m' or r.

If n denotes the number observed, n_0 — the true number of pairs having a certain value of m' and r,

$$n_0 = \frac{n}{p} \quad . \; . \; . \quad (3), \; p \text{ being given by } (2).$$

The function π can be determined from a comparison of the data for different groups of the apparent magnitude; for groups homogeneous with respect to the spectral type, i. e. containing equal percentages of each spectral subdivision, the relative frequency of the difference r must be the same whatever the apparent magnitude be; if nevertheless, as in table 1, the fainter stars show a deficiency of great r, this must be ascribed to the effect of the factor π.

The magnitude-groups of table 1 are *a priori* not fit for a direct comparison, the distribution of spectral types varying systematically with the apparent magnitude; the numbers for each spectral type within each group must be therefore multiplied by certain *reduction factors*, so as to make the percentage of a given spectral type equal to the average percentage of this type for all groups together. Table 2 gives the percentages and the corresponding reduction factors.

Table 2.
Reduction Factors.

	Sp. Type	Giants	B_0-B_9	A_0-A_5	F_0-F_8	G_0-G_5	K_0-K_5	M	Sum
	Percentage, all stars.	6.0	5.6	24.5	36.9	19.5	7.1	0.3	99.9
$\bar{m}_c=5.0$	Percent.	12.3	10.4	35.0	30.1	10.4	1.8	0	100.0
	Red. Fact.	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{4}$	2	4	0	—
$\bar{m}_c=7.0$	Percent.	4.5	5.1	23.6	44.1	17.5	5.1	0	99.9
	Red. Fact.	$\frac{4}{3}$	1	1	$\frac{5}{6}$	$\frac{10}{9}$	$\frac{7}{5}$	0	—
$\bar{m}_c=8.5$	Percent.	2.6	1.3	15.2	28.5	33.8	17.2	1.3	99.9
	Red. Fact.	2.3	4	1.6	1.3	0.6	0.4	0	—

Owing to the small number of the M-dwarfs, they were excluded from the discussion, so that the reduction factor 0 was attributed to them.

The numbers of table 1 were multiplied by the corresponding reduction factors and the data for all spectral types were joined together; table 3 gives the result.

Table 3.
Data reduced to the Average Percentages of the Spectral Types.
All types.

$r=m'-m$	0.0—0.4	0.5—0.9	1.0—1.9	2.0—2.9	3.0—3.9	4.0—4.9	5.0—6.9	7.0—8.9	9.0—10.9	11.0—13.0	Total
	Reduced Number										
$\bar{m}_c=5.0$	24.2	35.3	35.7	28.0	19.0	5.1	11.2	0.3	0.7	1.2	160.7
$\bar{m}_c=7.0$	112.7	76.9	85.5	28.2	16.1	5.6	0	0	0	0	325.0
$\bar{m}_c=8.5$	56.9	39.4	37.7	14.0	1.6	1.3	0	0	0	0	150.9
	Relative Number										
$\bar{m}_c=5.0$	1.000		0.600	0.471	0.319	0.086	0.188	—	—	—	—
$\bar{m}_c=7.0$	1.000		0.450	0.148	0.085	0.029	0.000	—	—	—	—
$\bar{m}_c=8.5$	1.000		0.392	0.146	0.017	0.014	0	—	—	—	—
	Effective Magnitude of the Fainter Component (m')										
$\bar{m}_c=5.0$	5.5		6.5	7.5	8.5	9.5	11.0	—	—	—	—
$\bar{m}_c=7.0$	7.5		8.5	9.5	10.5	11.5	12.5	—	—	—	—
$\bar{m}_c=8.5$	9.0		10.0	11.0	12.0	13.0	14.0	—	—	—	—

Taking the number from $r = 0.0$ to 0.9 as unity, the *relative numbers* of the table were obtained; the effective magnitude of the fainter component was obtained by adding to the combined magnitude, $\overline{m}_c$, the average difference, r; this is not quite correct, but of no significance in our case, since what we need here is a continuously varying argument of the function π, and for this purpose the adopted magnitudes can serve as well as the true ones.

From formulae (2) and (3) we have

$$n_0 \,.\, \pi(m') \,.\, \lambda(r) = n\,;$$

denoting the product $n_0 \,.\, \lambda(r)$ by ν, we obtain

$$\nu(r) \,.\, \pi(m') = n \quad . \; . \; . \quad (4)\,;$$

here ν is considered as depending on r only, which is true for the reduced numbers of table 3; substituting for n in (4) the relative numbers of table 3, the following equations were obtained:

$(\overline{m}_c=5.0)$	$(\overline{m}_c=7.0)$	$(\overline{m}_c=8.5)$
$\nu(0.5)\,.\,\pi(5.5)=1.000\pi(5.5)$	$\nu(0.5)\,.\,\pi(7.5)=1.000\pi(7.5)$	$\nu(0.5)\,.\,\pi(9.0)=1.000\pi(9.0)$
$\nu(1.5)\,.\,\pi(6.5)=0.600\pi(5.5)$	$\nu(1.5)\,.\,\pi(8.5)=0.450\pi(7.5)$	$\nu(1.5)\,.\,\pi(10.0)=0.392\pi(9.0)$
$\nu(2.5)\,.\,\pi(7.5)=0.471\pi(5.5)$	$\nu(2.5)\,.\,\pi(9.5)=0.148\pi(7.5)$	$\nu(2.5)\,.\,\pi(11.0)=0.146\pi(9.0)$
$\nu(3.5)\,.\,\pi(8.5)=0.319\pi(5.5)$	$\nu(3.5)\,.\,\pi(10.5)=0.085\pi(7.5)$	$\nu(3.5)\,.\,\pi(12.0)=0.017\pi(9.0)$
$\nu(4.5)\,.\,\pi(9.5)=0.086\pi(5.5)$	$\nu(4.5)\,.\,\pi(11.5)=0.029\pi(7.5)$	$\nu(4.5)\,.\,\pi(13.0)=0.014\pi(9.0)$.

The assumption was made that for $m' \leqslant 7.5$ the effect of the apparent magnitude is insensible, so that $\pi(5.5) = \pi(6.5) = \pi(7.5) = 1$. With this assumption the equations for $\overline{m}_c = 5.0$ and $\overline{m}_c = 7.0$ gave the following solution:

$$\nu(0.5) = 1.000\,;\ \nu(1.5) = 0.600\,;\ \nu(2.5) = 0.471\,;\ \nu(3.5) = 0.425\,;$$
$$\nu(4.5) = 0.274, \text{ and } \pi(8.5) = 0.750\,;\ \pi(9.5) = 0.314\,;\ \pi(10.5) = 0.200\,;$$
$$\pi(11.5) = 0.106 \quad . \; . \; . \quad (a).$$

Substituting for $\pi(9.0)$ the value $\dfrac{\pi(8.5) + \pi(9.5)}{2} = 0.532$ into the equations for $\overline{m}_c = 8.5$, the following supplementary values of π were obtained:

$$\pi(10.0) = 0.347\,;\ \pi(11.0) = 0.166\,;\ \pi(12.0) = 0.020\,;$$
$$\pi(13.0) = 0.027 \quad . \; . \; . \quad (b).$$

Through all values of π found a smooth curve was drawn. Fig. 1 represents this curve and the points on which the curve is based; in drawing the curve a double weight was attributed to the points of solution (a) in comparison with those of solution (b).

Table 4 contains the finally adopted values of π as read from the curve.

Table 4.

Coefficient of Selection, $\pi(m')$, depending on the Apparent Magnitude of the Fainter Component.

m'	$\leqslant$7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0
π	1.00	0.98	0.75	0.49	0.36	0.27	0.21	0.15	0.090	0.045	0.025	0.015
$\frac{1}{\pi}$	1.00	1.02	1.33	2.04	2.78	3.70	4.76	6.67	11	22	40	67

In table 1 the two last columns give the adopted effective magnitude of the fainter component, m', and the corresponding value of $\frac{1}{\pi}$; after multiplying the numbers of table 1 by this factor they will be freed from the effect of *selection in magnitude.* There will remain the selection depending on the difference in magnitude, r, represented by an unknown factor, $\lambda(r)$; it is difficult to say anything about the character of this function; generally we may expect that with the increasing r the discovery will grow more difficult, so that $\lambda(r)$ will decrease; in this case by freeing only from the selection in the magnitude we shall obtain a certain *minimal curve* for the distribution of the differences r, the number of great r being underestimated. However, from the following considerations it appears that the effect of selection depending on r is not as great as might be expected at the first glance. Let us imagine a number of pairs having the same magnitude of the fainter component, m'; the factor π will thus be constant, and only the function $\lambda(r)$ will represent the relative difficulty of discoveries; now r increases with the increasing brightness of the principal component; since the brighter stars are without doubt better examined in search of doubles than the fainter ones, the difficulty arising from the great difference in magnitude will be considerably counterbalanced by that circumstance; another factor favouring the discovery of companions to bright stars is their relative nearness and, consequently, their greater angular separation. From all these considerations it appears that in neglecting the factor $\lambda(r)$ we remain, nevertheless, not far from the truth, and that by the selection in the apparent magnitudes, $\pi(m')$, the greatest part of the entire selection may be accounted for.

The distribution $\chi(r)$ of the differences in magnitude was finally computed in the following way. Let $n_{i,k}$ denote the actually observed number having the difference in magnitude between r_i and r_k, $r_k > r_i$, and let $\pi_{i,k}$ be the corresponding average coefficient of selection; then

$$\nu_{i,k} = \frac{n_{i,k}}{\pi_{i,k}} \quad . \; . \; . \quad (5)$$

denotes the number freed from the effect of selection, or, as we will call it further, the *probable number*. Let us choose a quantity $P_{i,k}$, defined by

$$P_{i,k} = \frac{\nu_{i,k}}{\nu_{o,i}} \quad . \; . \; . \quad (6),$$

where $\nu_{o,i}$ is the sum of the probable numbers from $r = 0$ to $r = r_i$; $P_{i,k}$ may be called the *relative increment* of the probable number for the interval $r_i - r_k$; this quantity as well as $\nu_{i,k}$ may be regarded as determining the frequency-function χ.

From the data of table (1) three independent solutions for P or ν can be obtained, corresponding to the three different groups of the combined magnitude, m_c; to obtain a mean value, certain weights must be adopted for the single solutions; the weights were assumed proportional to

$$w_{i,k} = \pi_{i,k} \, . \, n_{o,i} \quad . \; . \; . \quad (7),$$

where $n_{o,i}$ denotes the number observed from $r = 0$ to $r = r_i$; formula (7) represents evidently a quantity proportional to the *a priori expected* observed number; it seemed not advisable to take the actually observed number as the weight, especially when it was small; e. g. a zero observed number would be entitled in this case to a zero weight, whereas obviously the weight is not less than for an observed number 1 or 2.

With the weight given by (7) the mean value of P from all groups was computed from

$$\overline{P}_{i,k} = \frac{\Sigma m_c \; P_{i,k} \, w_{i,k}}{\Sigma m_c \; w_{i,k}};$$

with the aid of (6) and (5) this expression is transformed into

$$\overline{P}_{i,k} = \frac{\Sigma_{m_c} \frac{n_{i,k} \cdot n_{o,i}}{\nu_{o,i}}}{\Sigma_{m_c} \pi_{i,k} \cdot n_{o,i}} \quad . \; . \; . \quad (8).$$

The summation denoted by Σ_{m_c} must be taken over the different values of the combined magnitude, m_c.

From the $\overline{P}_{i,k}$ the average probable number, $\overline{\nu}_{i,k}$, is determined by equation (6); relative numbers are obtained by taking a certain $\nu_{o,h}$ equal to 1. The frequency-function is determined by

$$\chi(\overline{r}) = \frac{\overline{\nu}_{i,k}}{r_k - r_i} \quad . \; . \; . \quad (9),$$

where $\overline{r} = \frac{r_i + r_k}{2}$. It is supposed that the interval $r_k - r_i$ is small enough.

From table 1 the values of χ were computed according to the method described; the result is contained in table 5.

3. Discussion of Results.

From a comparison of the data for different spectral types we may conclude that the general character of the function χ is practically the same for all spectral types, except B_0—B_9; considerable divergences appear only where the actually observed number is small and where therefore the data are of small weight. Since the spectral subdivision is equivalent to a subdivision according to the absolute magnitudes, the behaviour of the function χ indicates that the first case considered in § 2 takes place, when $\frac{\partial \chi}{\partial M'} = 0$ and when χ depends on r only. The exception for the B — stars seems to add another peculiarity to the many which these stars exhibit. The omission of the selection in the difference, $\lambda(r)$, could have no influence on the chief result obtained — that the distribution is a function of r only; the true distribution would be equal to $\frac{\chi(r)}{\lambda(r)}$ and depend thus also upon r only.

On Fig. 2 the data of table 5 are represented graphically. The curve for *all spectra* has no resemblence to a Gaussian

Table 5.

Frequency-Function, $\chi(r)$, of the difference in Magnitude for the Pairs in Motion.

Spectrum of Bright. Comp.		Mean Difference in Magnitude ($\overline{r}$). 0.2	0.7	1.45	2.45	3.45	4.45	5.45	6.45
Giants (F_8—M)	χ	2.00	1.58	1.09	0.69	1.46	0.60	1.24	3.44
	n	10	8	8	4	5	1	1	1
B_0—B_9	χ	2.00	2.50	1.75	0.80	0.29	0	0	0
	n	8	11	12	4	1	0	0	0
A_0—A_5	χ	2.00	1.34	1.20	0.60	0.69	0.42	0.32	0.00
	n	49	31	46	16	11	3	1	0
F_0—F_8	χ	2.00	1.34	1.02	0.83	0.70	0.59	0.96	0.00
	n	81	51	57	27	13	5	3	0
G_0—G_5	χ	2.00	2.12	1.34	0.74	0.69	0.23	0.93	5.74
	n	40	32	30	13	7	1	1	2
K_0—K_5	χ	2.00	1.38	0.90	0.96	1.24	1.00	0	0
	n	17	11	9	5	3	1	0	0
All Spectra except B_0—B_9	χ	2.000	1.506	1.120	0.793	0.836	0.495	0.750	1.092
	$\log \chi$	0.301	0.178	0.049	$\bar{1}.899$	$\bar{1}.922$	$\bar{1}.695$	$\bar{1}.875$	0.038
	n	197	134	150	65	40	11	6	3

n denotes the *actually observed* number.

distribution, which should be expected, if the distribution of luminosities of the components of double stars were identical, or at least analogous to the distribution of luminosities of single stars. The curve has a minimum at about $r = 4.5$ and increases on both sides from this value; the most frequent difference is 0.0, indicating that in the origin of double stars the conditions are favourable for the formation of components of equal brightness. The increase of χ from $r \geqslant 4.5$ is without doubt real, although based on a relatively small observed number; the discovery of such faint objects as the companions of *Sirius* and *Procyon* indicates that a large number of double stars with the difference in brightness over 10 magnitudes must exist; if only few of this kind are known, it is because of the exceptionally difficult conditions of observation.

Our result as to the general character of the curve indicates that components of *close* double stars cannot be regarded, from the statistical point of view, as representatives of single stars; in counting them together we introduce consciously a non-homogeneity which can, e. g., considerably disfigure our conclusions on the Luminosity-Curve of the stars in the nearest neighbourhood of the sun. The probability of a member of a close double system to have a certain luminosity is not an independent function of this luminosity, but depends on the luminosity of the other component.

The other conclusion which may be drawn from the standpoint of stellar evolution is that components of double systems differing in magnitude seem to change their magnitude with the time with equal speed, or that the "rate of cooling"[1] is on the average equal for the brighter and the fainter component; were it not so, the frequency-function $\chi(r)$ would show a systematical change with the progression of spectral type, provided the "later" spectral types represent, on the average, stars that were subject to evolutionary changes during a longer interval than the "earlier" types. However, this conclusion can be accepted only with certain reserves; the spectral type, or — which is practically the same — the absolute magnitude of a star may be assumed as depending upon the following factors: 1) an invariable parameter, by which the conditions at the beginning of the evolutionary course are determined; *on the average* the mass may be assumed as such a parameter; a quantity, called the "*Initial Magnitude*", was introduced in *T.P.* 25_2 for the same purpose; 2) the time elapsed since the beginning of evolution; 3) the "rate of cooling" or the function determining the decrease of luminosity with the time; this function way be assumed as depending on the first parameter and on the luminosity. Within a group of stars having the same luminosity there will be members of different ages: very old stars with a high Initial Magnitude, and stars which just began their evolution with a low value of the Initial Magnitude; therefore the difference in spectral type or luminosity is only partly due to the relative age, the other part — and, probably, the greater one — being produced by differences in the initial conditions. The difference in the relative age of different

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1 See *T.P.* 25_2, p 5 and foll.

spectral subdivisions must be, indeed, small; the masses derived by Fr. H. Seares[1]) from the hypothetical parallaxes of Jackson and Furner show such a great variation with the spectral type that the difference in the mass alone, with a corresponding difference in the inner structure and the „activity of matter", may account for the observed difference in the luminosity. On the other hand, if we take as a working hypothesis the "rejuvenation theory"[2]) exposed in *T.P.* 25_2, with the law of "radioactive cooling" (pp. 21, 22), the average decrease in magnitude will be equal to $\frac{1}{k}=\frac{v}{q}$, v representing the *rate of cooling* and q — the *frequency of catastrophes*; from a comparison with the Luminosity-Curve of Kapteyn and Van Rhijn we found $k=0.6$ (loc. cit. p. 22), which corresponds to an average decrease during a star's "life-time" of about 1.7 magnitudes (bolometric); the visual range must be somewhat greater, say, about 2 magnitudes. This is the average difference due to age, between the oldest and the youngest stars; the actual range from the *A*-stars to the *K*-dwarfs is about 8 magnitudes, so that only $\frac{1}{4}$ or even less of the total difference in luminosity of the different spectral classes can be attributed to the difference in the age, the other $\frac{3}{4}$ being explained by the difference in the initial conditions. From this standpoint the likeness of the function χ for different spectral classes is not surprising; our conclusion as to the constancy of the rate of cooling, v, for stars of different luminosity must be altered in this respect that *a very great* dependence of v upon the luminosity is improbable, whereas a moderate variation of v cannot be revealed by our data; for instance the following figures may be given: for a total range in the age corresponding to a decrease of 1—$1^1/_2$ magn. and a difference in the magnitudes $=3$, the difference in the decrease must be less than $\frac{1}{2}$ magn.; thus a change in the absolute luminosity in the ratio 16:1 produces a difference in the rate of cooling less than in the ratio 2:1; in other words, if the dependence of the rate of cooling upon the luminosity, L, can be represented by a formula like

$$v=c\,L^a \quad . \; . \; . \quad (10),$$

1) Astrophysical Journal, **55**, p. 179; Mt. Wilson Contrib. **226**.

2) I quote this term from a letter of H. N. Russel.

the absolute value of the exponent a must be less than $\frac{1}{4}$, $|a| < \frac{1}{4}$.

It may be remarked that the special choice of the "radioactive" law of cooling has only a small influence on the result obtained; any other reasonable law of cooling would lead practically to the same result; the chief assumption lies in the adopted difference in the "age" of different spectral classes; this difference is assumed small enough, and if it is underestimated, the upper limit for a will be overestimated. Thus we may assume that a is very small. It is interesting to compare this result with the "gravitational law of cooling", discussed in *T.P.* 25_2, pp 25—31; formula (28), loc. cit. p. 27, gives

$$\log v = C - \frac{1}{3}(M - M_0),$$

M_0 being the Initial Magnitude, and table 10, loc. cit. p. 29, gives approximately

$$C = \text{Const.} - 0.15\, M_0;$$

adopting, for an average star, $M_0 = M - \text{const.}$, we obtain

$$\log v = \text{const.} - 0.15\, M,$$

which corresponds to $a = \frac{0.15}{0.4} = 0.4$ approximately; this value is above our upper limit; but the difference increases if the „gravitational law of cooling" is applied to the estimate of the upper limit of a; this law leads to an average decrease of magnitude during a star's life-time of about 4 magnitudes, or the double of the value adopted previously; thus in this case the upper limit of a comes out only $\frac{1}{2}$ of the formerly found, or $|a| < \frac{1}{8}$. From all these considerations it appears that *the frequency-function of the differences in magnitude of components of close double stars can hardly be reconciled with the gravitational law of cooling;* the variation of the rate of cooling with the luminosity must be much slower than required by this law, if such a variation exist at all; on the contrary, the "radioactive law of cooling", or *a uniform variation of the absolute magnitude with the time is in good agreement with the observational data,* since it requires that the difference in magnitude of the components remains constant during the whole time of evolution.

A further conclusion is that the distribution of the luminosities between the components of close pairs obeys a certain law equal for all stars of moderate mass, the very massive *B*-stars showing a peculiar distribution: the most frequent are two extremities — whether a near equality, or a very great difference in the brightness; the cause of the distribution must be in the early period of the life-time of the binary.

Let us suppose that the close pairs discussed here originated through fission[1]); then the relative luminosity of the components will be on the average a function of the relative mass, the more massive being the more luminous. This statement is confirmed by the majority of binaries with known mass-ratio, although certain exceptions exist[2]); from 19 pairs with known mass-ratio E. Bernewitz derives the following *average* correlation between the mass-ratio, $\mu_2 : \mu_1$, and the difference in magnitude, r[3]):

Table 6.

r	0.0	1.0	2.0	3.0	4.0	5.0	11.5
$\mu_2 : \mu_1$	1.00	0.90	0.81	0.74	0.68	0.62	0.36
$\log \mu_2 : \mu_1$	0.000	—0.046	—0.092	—0.131	—0.168	—0.208	—0.444

The last figure is not given by Bernewitz, but represents the mean of the two pairs with the greatest magnitude-difference known, *α* Canis Majoris and *α* Canis Minoris.

The data of the table can be very satisfactorily represented by the formula

$$\log (\mu_2 : \mu_1) = -0.040\, r \quad \ldots (11);$$

this corresponds to the following relation between the ratio of the masses, $\mu_2 : \mu_1$, and the ratio of the luminosities, $J_2 : J_1$:

$$\frac{J_2}{J_1} = \left(\frac{\mu_2}{\mu_1}\right)^{10} \quad \ldots . (12).$$

The extraordinarily high value of the exponent in formula (12) contadicts all we know of single stars; from the average

1) The fission theory is thoroughly discussed by H. N. Russel: *On the Origin of Binary Stars,* Astrophysical Journal **31** (1910) pp. 185—207, and by J. H. Jeans: *Problems of Cosmogony and Stellar Dynamics,* Cambridge Univ. Press, 1919.

2) The most striking one is 85 Pegasi, where the by 5 mg. fainter companion is 2,6 times more massive.

3) Astr. Nachrichten, **5089** (1921). A very similar correlation was derived by the writer from a smaller number of pairs: Astrophysical Journal **44** (1916), p. 297.

masses of the *principal* components of double stars determined by Fr. H. Seares[1]) we found the following approximate linear relation between the absolute magnitude, M, and $\log \mu$:

$$\log \mu = -\tfrac{2}{15} M + \text{const} \quad \ldots \quad (13),$$ whence the following dependence of the luminosity upon the mass for the dwarf series from A_0 to M results:

$$J = \mu^3 \times \text{const.} \quad \ldots \ldots \quad (14).$$

Thus the variation of the luminosity with the mass is much more rapid for the fainter than for the brighter components of binaries; the latter may be regarded as probably representing ordinary stars, whereas the former represent evidently exceptional conditions. A small part of the difference may be due to selection but the major part is doubtlessly real. The following explanation of the strange behaviour of the luminosities of the fainter components may be suggested. Let us suppose that the energy of the stars is supplied by a certain kind of *active matter*, and that this active matter had the tendency to condense near the centre of the original nebula more closely than the remaining mass; then, in the process of fission, the smaller mass will originate from the outer parts of the nebula and will thus carry away a smaller proportion of the active matter than the proportion remaining in the primary component, since the latter will retain the greatest part of the active nucleus; the total luminosity of a star may be assumed proportional to 1) the mass, 2) the degree of activity and 3) the proportion of active matter (the amount per unit mass); for ordinary stars, as well as for binary systems as a whole, bodies of different mass contain probably on the average the same proportion of the active matter, so that the main variation of the luminosity with the mass must be attributed chiefly to the first two factors; the degree of activity, which from all appearences is highly depending on the central temperature and, consequently, on the mass[2]), is evidently the most important factor for ordinary stars; formula (14) would indicate a variation of the activity proportional to μ^2. However, for close double stars originating by fission the most important

1) Loc. Cit. p. 26, Table IX.

2) Compare the discussion of this question by A. S. Eddington; Zeitschrift für Physik, 7 (1921) p. 395; also *T.P.* 25_2 (1922), pp. 38—43, where the writer independently arrived at substantially analogous conclusions.

factor determining the luminosity seems to be the *proportion* of the active matter; from a comparison of formulae (12) and (14) we may infer that this proportion in the fainter component must vary with the 7^{th} power of the mass-ratio; this indicates a high degree of concentration of the active matter in the nebula before fission occurred.

The above considerations give another reason why the fainter companions of double stars cannot be regarded as representatives of normal stars; the opinion has been expressed from many sides that Kapteyn's luminosity-curve gives too low a number of very faint stars, the ground for this opinion being the relative frequency of discoveries of absolutely very faint stars; it is useful therefore to take into consideration that such bodies as the companions of *Sirius* and *Procyon,* perhaps also *Proxima Centauri,* cannot serve for purposes of such a criticism; these exceptional objects are faint because the central luminous body has probably retained almost the whole amount of the active matter of the system; in dealing with the luminosity distribution of stars only the *combined luminosity* of the whole system must be regarded as equivalent to the luminosity of a single star, the distribution of luminosities between the single components following a peculiar law which has nothing in common with the general frequency-function of stellar luminosities.

4. Luminosity-Curve of Distant Companions.

About 17 % of the close pairs discussed above have, according to Burnham's General Catalogue, distant companions, which in the majority of cases seem to be physically connected with the close pair, forming thus triple or multiple systems; it seemed interesting to investigate the distribution of luminosities of these distant components; owing to the small number of objects — a total of only 121 were used — the discussion may be regarded as preliminary; we hope that an investigation based on the whole material contained in Burnham's catalogue will follow.

In the counts were included companions for which the probability of optical vicinity was less than about $\frac{1}{100}$; for this purpose the following maximum distance for the companion was adopted:

Magn., β. G. C.	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
Max. Distance	400″	250″	150″	100″	50″	33″	22″	15″	12″.

Since the total number of pairs was 645, the probable number of optical companions included in the counts must be less than 1 % or about 6; so that not more than 1 in 20 of the stars counted below will be in optical vicinity; since the probability of a companion to be optical is equal for the bright and faint ones, the false companions must be uniformly distributed over all magnitudes and the resulting Luminosity-Curve will thus be affected by them but insensibly.

Table 7 gives a list of the distant companions contained in Burnham's General Catalogue, used or rejected in our counts.

Table 7.

Distant Companions of Pairs in Motion. $m_2 - m$ = difference in magnitude between the distant companion and the brighter component of the pair in motion.

1) Companions Used in the Counts.										2) Companions Rejected	
β G.C.	$m_2 - m$	β G.C.	$m_2 - m$	β G.C.	$m_2 - m$	β G.C.	$m_2 - m$	β G.C.	$m_2 - m$	β G.C.	$m_2 - m$
1070[1])	—2.0	10363	8.6; 6.9	2883	6.3; 3.5	8909	3.3	1262	3.9	1036	8.6; 8.0
1471	4.5	10533	3.7	„	2.3; 3.0	8986	—0.5	1457	2.1	2381	7.0
1761	3.4	10643	1.4	2902	8.0	9450	0.7	2109[4])	+5.1	5951	6.5
2093	0.5; 1.0	10829	5.7	3146	—2.3	9459	0.8	3291	1.7	6243	11.5; 8.6
3559	2.2	11761	0.2	3402	—0.5; +7.0	9782	1.8	3382	—2.3	830	7.2
3876	3.9	12404	2.1	3542	5.2; 5.0; 3.1	10281	—0.5; +4.5	3962	0 2	4828	6.0
4122	6.8	12701	2.5	„	3.0; 3.5	10423	1.3	3990	5.7; 5.2; 3.0	5224	4.3
4414	2.5	70	2.7	3678	7.9	10487	—1.4	4821	4.0	7885	9.6; 8.1
4477[2])	0.5	440	2.2	4098	4.8; 5.3	10607	4.5	4941	2.3	7914	12.0
4771[3])	3.8; 8.5	614	5.0; 4.2	4828	5.2	10709	—1.8	5397	1.5	10607	6.5
5235	6.5	„	3.5; 1.9	5841	5.6	11125	4.4	6046	1.3	5397	7.2
5951	7.2	941	2.3	6758	1.5	11214	7.5; 2.2	6571	0.9		
6296	4.0	946	0.0	7040	4.0	11690	1.9	8162	—6.7		
6842	—1.6	1002	1.3	7490	0.2	11715	6.9; 7.9	8736	1.4		
7487	2.3	1501	5.3	7495	3.6; 4.0	11968	1.2	9090	2.4		
7563	7.5; 5.5	1559	4.8	7739	3.7	12036	4.7	10044	3.2		
7878	8.0	2279	2.4; 8.2	7751	3.5	12510	5.1	10727	5.3		
7929	4.5	2544	—2.5	7914	7.6	12563	—1.2	11542	—0.8		
9114	1.0	2588	6.5	8167	0.5; 0.8	12709	3.5	12731	1.6		
9643	3.2	2857	2.4; 5.7	8574	2.4			11222	6.0		

1) γ Andromedae 2) ζ Cancri 3) ε Hydrae 4) 0_2 Eridani; difference taken with opposite sign.

Table 8 represents the distribution of the differences in magnitude according to the spectral type.

The material is too scarce to allow of a separate treatment of the spectral types. From the general character of the distribution the impression was obtained that the luminosities of the distant companions have a distribution independent of the luminosity or spectral type of the close pair, and on this assumption the reduction was made; it corresponds to the second case of § 2, when $\frac{\partial \chi}{\partial r} = 0$, and when the true Luminosity-Curve can be found without fearing the influence of the selection in the primary components.

Of the two factors of selection occurring in formula (2) the factor $\lambda(r)$ can be put equal to 1, since the difference in magnitude hardly affects the conditions of discovering distant companions, so that only the magnitude-factor, $\pi(m_2)$, remains; from the small number of the observational data an independ-

Table 8.

Distribution of Differences in Magnitude of the Distant Companions of Pairs in Motion.

m = magnitude of the brighter component of the close pair;
m_c = combined magnitude of the close pair;
m_2 = magnitude of the distant companion;
the spectral type refers to the close pair, except o_2 Eridani.

$m_c \leqslant 6.00$; $\overline{m}_c = 5.0$.

Sp. Type	Giants	B_0-B_9	A_0-A_5	F_0-F_8	G_0-G_5	K_0-K_5	M	Sum	$1:\pi$
m_2-m	Number								
< 0.0	0	1	1	0	0	0	0	2	1.00
0.0 . . . +0.9	0	0	0	0	1	0	0	1	1.00
1.0 . . . 1.9	0	1	1	1	0	0	0	3	1.00
2.0 . . . 2.9	0	1	5	3	1	0	0	10	1.00
3.0 . . . 3.9	1	6	3	0	0	0	0	10	1.00
4.0 . . . 4.9	1	0	1	2	0	1	0	5	1.00
5.0 . . . 5.9	0	2	3	3	0	0	0	8	1.00
6.0 . . . 6.9	0	1	2	2	0	0	0	5	1.33
7.0 . . . 7.9	2	1	0	2	0	0	0	5	2.78
8.0 . . . 8.9	1	1	0	3	0	0	0	5	4.76
$\geqslant 9.0$	0	0	0	0	0	0	0	0	—
Total	5	14	16	16	2	1	0	54	—

Table 8. Continued.

$6.01 \leqslant m_c \leqslant 7.99$; $\overline{m}_c = 7.0$									
Sp. Type	Giants	B_0-B_9	A_0-A_5	F_0-F_8	G_0-G_5	K_0-K_5	M	Sum	$1:\pi$
m_2-m				Number					
< 0.0	0	0	3	0	1	0	0	4	1.00
0.0 . . . +0.9	0	0	0	2	1	0	0	3	1.00
1.0 . . . 1.9	1	0	2	6	3	0	0	12	1.00
2.0 . . . 2.9	0	0	1	2	2	0	0	5	1.00
3.0 . . . 3.9	0	1	0	5	2	0	0	8	1.00
4.0 . . . 4.9	0	1	1	4	0	0	0	6	1.33
5.0 . . . 5.9	1	2	1	1	0	1	0	6	2.78
6.0 . . . 6.9	0	0	1	0	1	0	0	2	4.76
7.0 . . . 7.9	0	0	0	1	1	0	0	2	11.0
$\geqslant 8.0$	0	0	0	0	0	0	0	0	—
Total	2	4	9	21	11	1	0	48	—

$m_c \geqslant 8.0$; $\overline{m}_c = 8.5$									
Sp. Type	Giants	B_0-B_9	A_0-A_5	F_0-F_8	G_0-G_5	K_0-K_5	M	Sum	$1:\pi$
m_2-m				Number					
< 0.0	1	0	0	0	3	0	1	5	1.00
0.0 . . . +0.9	0	0	3	1	0	2	1	7	1.00
1.0 . . . 1.9	0	0	0	0	2	0	0	2	1.00
2.0 . . . 2.9	0	0	0	1	1	0	0	2	1.02
3.0 . . . 3.9	0	0	0	0	1	0	0	1	2.04
4.0 . . . 4.9	0	0	0	0	1	1	0	2	3.70
$\geqslant 5.0$	0	0	0	0	0	0	0	0	—
Total	1	0	3	2	8	3	2	19	—

ent derivation of π for the distant companions seemed impossible; on the other hand, this factor could not be assumed equal to the values found for the close pairs; the limiting magnitude of the distant components was from 2.5 to 4 magnitudes below the limiting magnitude of the faint components of the close pairs which indicates more favourable conditions of observation of the former; therefore the values of π were assumed the same as given in table 4, only with a shift of the argument by 3 magnitudes, so that for $m_2 = m' + 3.0$, $\pi(m_2) = \pi(m')$. The last column of table 8 gives the adopted values of $1:\pi$.

The absolute magnitude of the distant components could be estimated fairly well from the spectral type of the brighter component of the close pair and the difference $m_2 - m$; the following values were adopted as average absolute magnitudes of the different spectral subdivisions:

Spectral type	Giants	B_0—B_9	A_0—A_5	F_0—F_8	G_0—G_5	K_0—K_5 and M
Abs. Magn. adopted (π=1″)	—4.0	—6.0	—4.0	—2.0	0.0	+2.0.

By adding the difference m_2—m to these figures, the absolute magnitude of the distant companions was obtained. In other respects the derivation of the frequency-function of the absolute magnitudes of the distant companions was made almost exactly in the same manner as the derivation of the distribution of the differences in magnitude for the close pairs; the only difference was that every spectral type formed a separate group for which the quantities P were determined (see form. (6)), and the sums of form. (8) were taken not only over the different groups of m_c, but also over the different spectral subdivisions. The final result is contained in table 9.

Table 9.

Luminosity-Curve for the Distant Companions of Close Pairs.
M = absolute magnitude, π = 1″.
$\varphi(M)$ = Luminosity-Curve.
n = observed number.

M	−4.5	−3.5	−2.5	−1.5	−0.5	+0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	Total
$\varphi(M)$	1.00	1.14	2.89	2.65	4.51	3.96	6.54	9.48	8.30	4.57	9.20	40.8	64.0	—
log φ (M)	0.00	0.06	0.46	0.42	0.65	0.60	0.82	0.98	0.92	0.66	0.96	1.61	1.81	—
n	1	4	11	10	15	12	16	17	9	4	3	6	2	110

On fig. 3 the result is represented graphically; for comparison the Luminosity-Curve derived by Kapteyn and Van Rhijn [1]) for the stars as a whole is given. A great difference exists between both curves, although as an interesting circumstance the coïncidence of the most pronounced secondary maximum of our curve at M=+2.5 with the maximum of Kapteyn's curve may be noted. The general character of the Luminosity-Curve of the distant companions seems to be an almost linearvariation of log $\varphi(M)$ with M and a relatively greater number of faint stars than for Kapteyn's Luminosity-Curve. It is not advisable, however, to draw very detailed conclusions from our preliminary results.

1) Mt. Wilson Contributions, 188.

5. Summary.

1. The frequency-distribution of the differences in magnitude, $\chi(r)$, for 645 close pairs showing relative motion and the frequency-distribution of the absolute magnitudes, $\varphi(M)$, for 121 distant companions of the close pairs is derived and discussed.

2. For all spectral types except B_0—B_9 the distribution of the differences in magnitude of the close pairs is practically the same; this indicates a close statistical relationship between the magnitudes of the components of close pairs, a relationship which is decidedly in favour of the fission-theory.

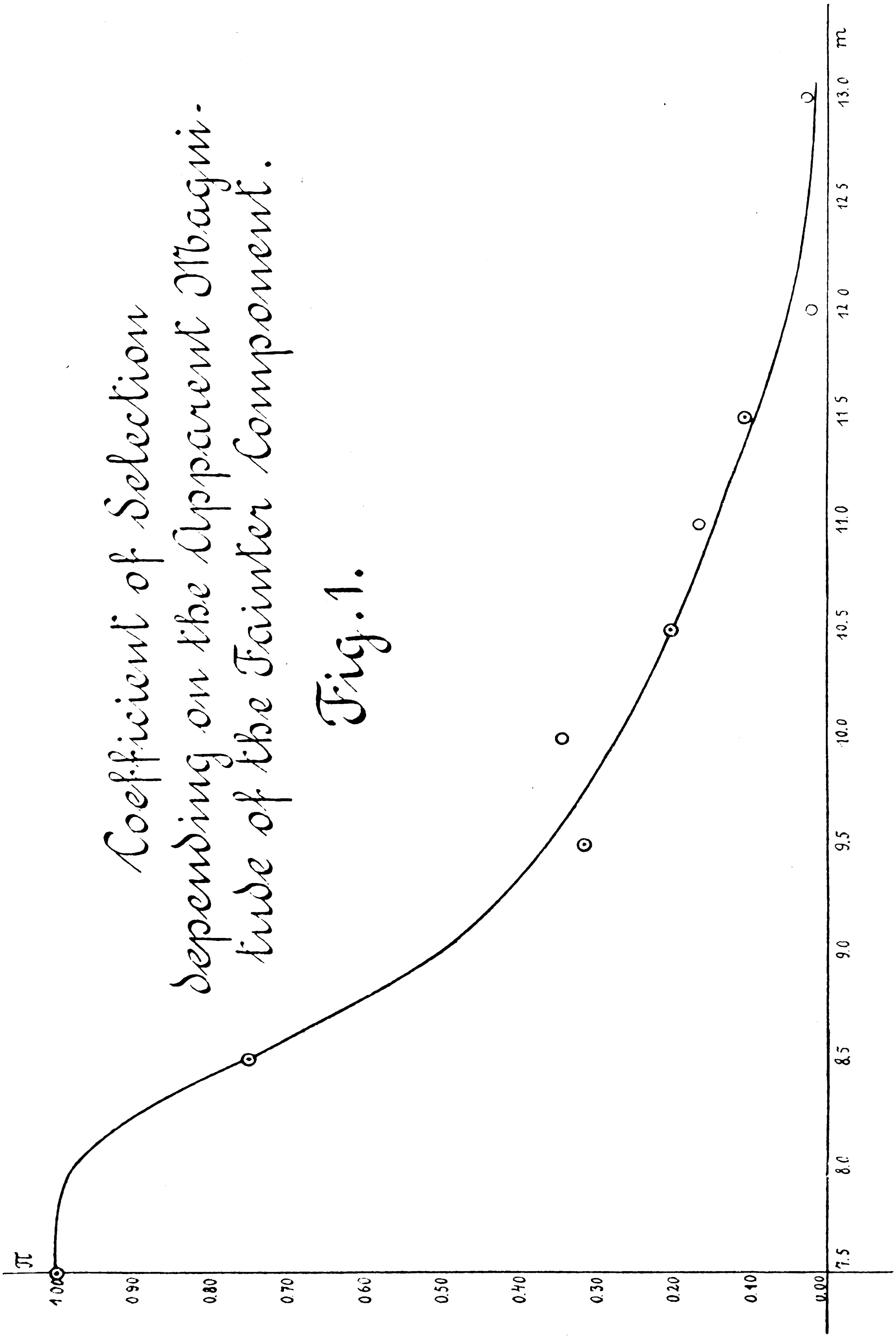
3. The near likeness of $\chi(r)$ for different spectral types indicates that the "rate of cooling" or the rate of the supposed decrease of the luminosity with time must be nearly equal, or vary very little with the luminosity or mass; in any case, the variability is less than required by the theory of gravitational contraction. Attention is drawn to the circumstance that the differences in spectral type or luminosity can only partly be due to the differences in age, and that the chief factor determining the spectral type lies without doubt in the initial conditions: the mass, the amount of "active matter", etc. From this standpoint conclusions on the relative speed of evolution based merely on the difference in the spectral type are by no means justified; a certain difference in the spectral type, or luminosity, establishes itself from the beginning and remains probably constant, the stars advancing in their evolution with equal speed.

4. Whereas the luminosity of ordinary dwarfs varies approximately with the 3-d power of the mass, the relative luminosity of the components of close pairs varies with the 10^{th} power of the mass-ratio. Such a high dependence of the luminosity on the mass may by explained on the assumption that in the nebula from which the pair originated by fission the *active matter*, which is supposed to be the source of the stellar energy, formed a nucleus of great concentration, so that in the case of fission into unequal masses the disparity in the amount of active matter was much greater than the disparity in mass.

5. The distribution $\chi(r)$ for the close pairs bears no resemblence to a Gaussian distribution nor to the general Luminosity-Curve of the stars; the peculiar form of the curve may

give information on the statistical laws accompanying the process of fission, or on the relative frequency of different mass-ratios.

6. Components of close pairs cannot be regarded as representants of ordinary stars; in the statistics of stellar lumino-



Coefficient of Selection depending on the Apparent Magnitude of the Fainter Component.

Fig. 1.

sities the combined luminosity of such pairs must be regarded as an equivalent to the luminosity of single stars; especially in counting the faint companions as separate individuals the result may be entirely disfigured. As to the brighter components, they are probably much more near the normal conditions.

7. The distribution of luminosities of the distant companions of close pairs seems to show a character independent of the spectrum or luminosity of the close pair; the Luminosity-Curve found has to some extent a character intermediate between the distribution $\chi(r)$ for the close pairs and Kapteyn's Luminosity-Curve; the distribution resembles somewhat the distribution of luminosities in globular clusters, especially in what concerns the *secondary maximum*, although the slope of the curve representing log $\varphi(M)$ as the function of M is much less than for fhe globular clusters.

Before concluding I should like to express my thanks to Miss W. Kosenkranius and Mr. R. Livländer whose assistance was most helpful to me in the preparation of this paper.

May, 1923.

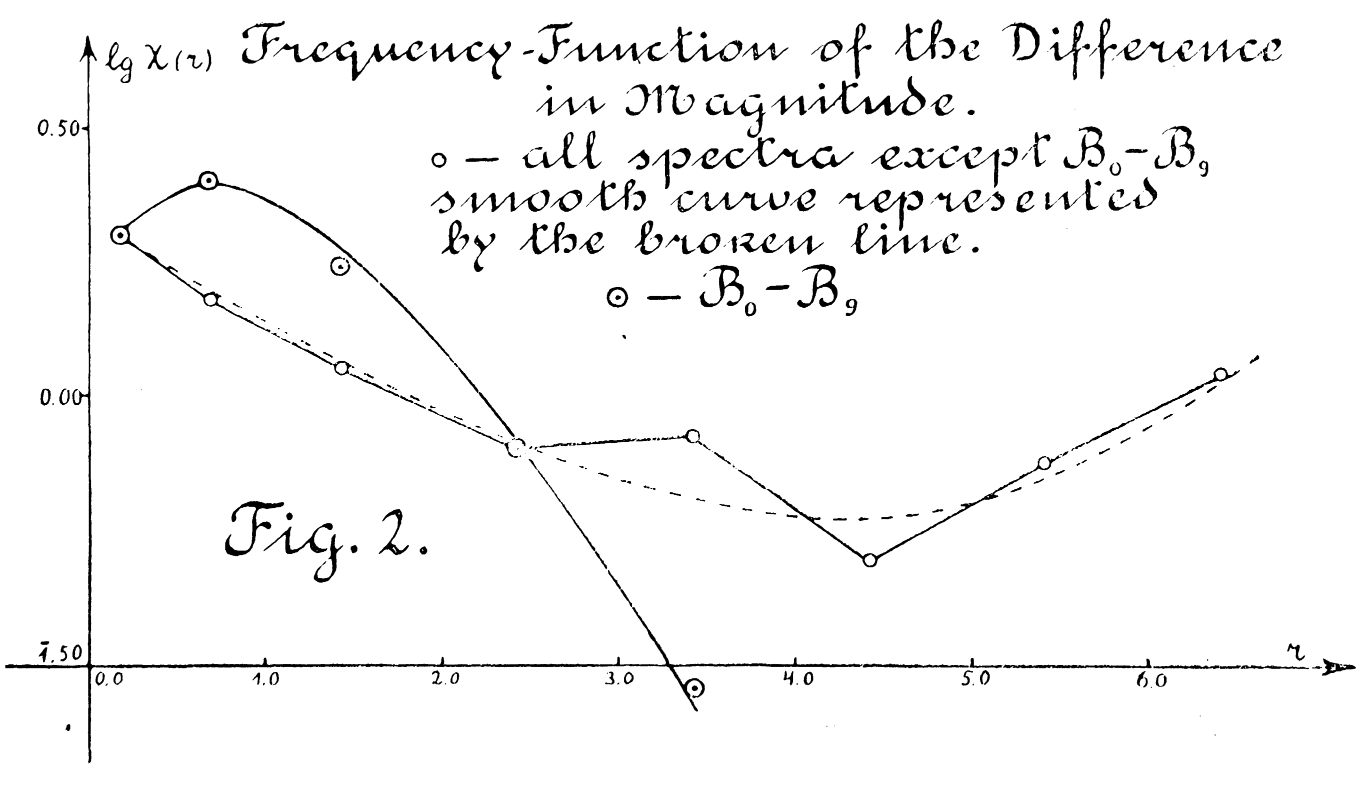


Fig. 2.

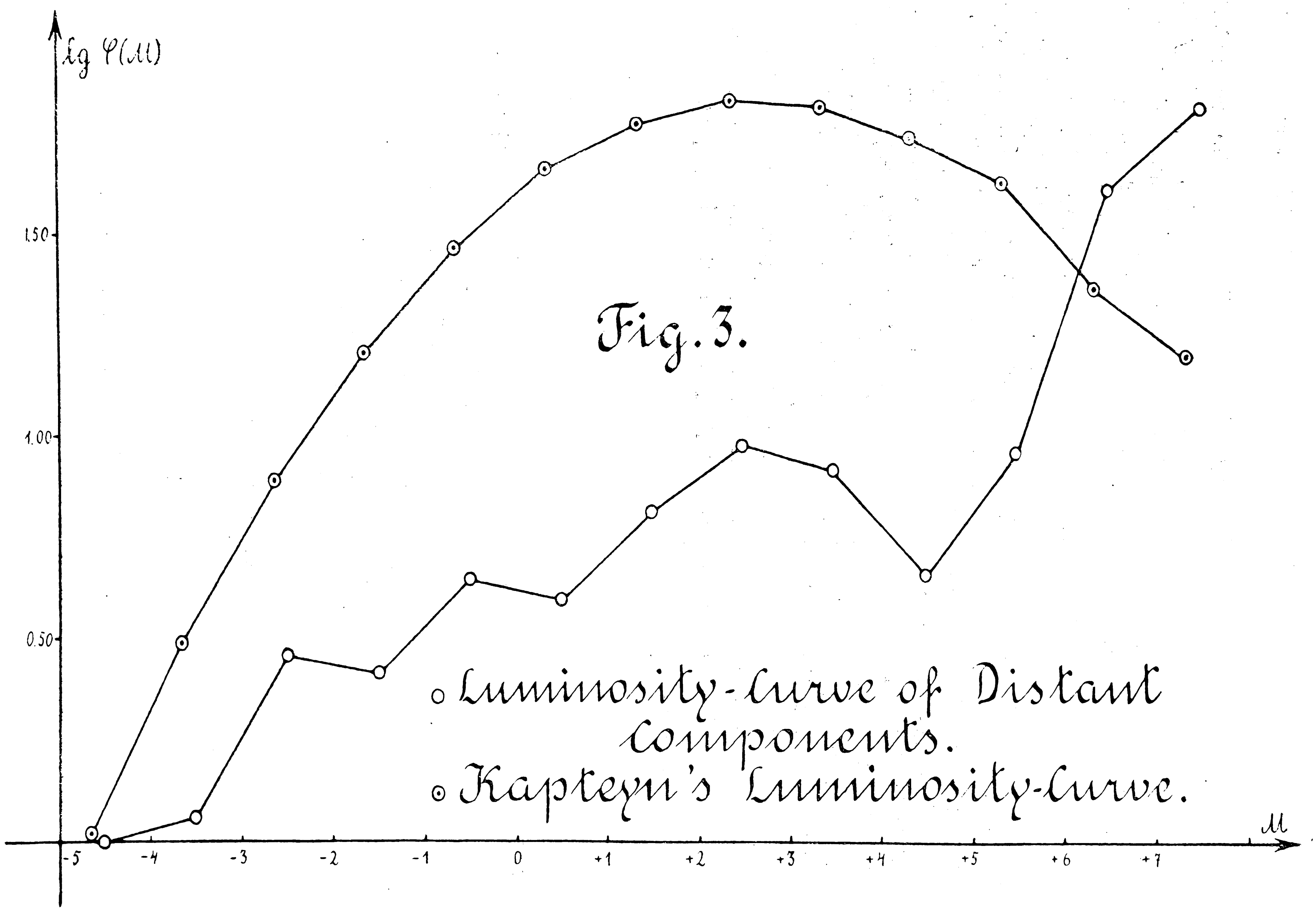

lg φ(M)
1.50
1.00
0.50
-5
-4
-3
-2
-1
0
+1
+2
+3
+4
+5
+6
+7
M
Luminosity-Curve of Distant Components.
Kapteyn's Luminosity-Curve.

Fig. 3.