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PHOTOGRAPHIC OBSERVATIONS OF THE BRIGHTNESS OF NEPTUNE. METHOD AND PRELIMINARY RESULTS

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K. Mattiesen, Tartus.

1. Introduction.

These measurements were undertaken on the Tartu (Dorpat) observatory with the chief purpose to detect any variation in the brightness of the planet, if such a variation exist at all; in the case of a positive result a clue for the determination of the period of rotation of Neptune would be put into our hands. The idea is not new: after the announcement of Maxwell Hall[1]) the question of the variability of Neptune was discussed many times, but no decisive results were obtained[2]). As the main result of the numerous observations of E. C. Pickering, G. Müller and J. M. Baldwin appeared the conclusion, that if any variability exist, its amplitude must be small — say, of the order of $0^{mg}.1$ or even less; if so, the method of visual photometry is unable to solve the problem, and more refined methods are required. Since Neptune, according to P. Guthnick, seems to be beyond the reach of the photoelectric cell, the best method fit for our purpose is the method of extrafocal photographs.

Variations in the light of a planet depending on rotation can be expected à priori; in the case of Mars variations of the order of $0^{mg}.1$—$0^{mg}.15$ were observed since 1914 by P. Guthnick[3]) and, independently, by the writer[4]); for Jupiter and Saturn were found by Guthnick irregular variations, which generally could not be made to agree with the period of rotation and had probably their source in certain disturbances occurring in the atmospheres of these planets; in 1920, however, Jupiter manifested a variability of $0^{mg}.14$ of a period equal to the period of rotation[5]). From these observations variations in the brightness

1) Monthly Notices **44**, p. 257; see also M. N. **75**, p. 626.

2) For reference see Photometric Measurements of Neptune, January to April 1908, by J. M. Baldwin. Monthly Notices **68**, p. 614.

3) P. Guthnick und R. Prager. Photoelektrische Untersuchungen an spektroskopischen Doppelsternen und an Planeten. Veröffentlichungen d. Kgl. Sternw. Berlin-Babelsberg. B. I, H. 1 (1914) and B. II, H. 3 (1918).

4) Zum Lichtwechsel des Planeten Mars. Astronomische Nachrichten 5162.

5) P. Guthnick. Veränderlichkeit der Helligkeit des Jupiter in der Opposition 1920. Astronomische Nachrichten 5067, p. 39.

of the planets seem to be a general rule; the success of the measurements of Mars made by the writer indicates that the approaching of the problem with the method of extrafocal photographic photometry is not a hopeless task, though in the case of Neptune, owing to the faintness of this planet, the difficulties are greater than for Mars.

The present paper deals chiefly with the method of investigation; as a preliminary result a variability in the photographic brightness within a range not surpassing $0^{mg}.15$ has been found; the material is, however, too scant to allow of any reliable derivation of the period of variation; the relatively small number of plates obtained is due to the unusually bad weather conditions during the period of observation — spring 1922.

On the contrary, the average photographic brightness of Neptune during the period of observation can be determined with high precision; a definitive result, however, cannot be given here, for the magnitudes of the comparison stars on an absolute scale are known with a smaller degree of precision than the result of our measurements; our preliminary scale of magnitudes is based on 2 stars of the Göttingen Actinometry[1]); for the definitive result a special determination of the magnitudes of all comparison stars on an absolute scale is needed.

The observations of the photographic brightness of Neptune will be systematically continued here; since definitive results cannot be expected in a very short interval of time, it has been thought that the publication of our preliminary results must not be delayed; the aim was, besides, that more attention may be drawn to the determination of the photographic brightness of Neptune[2]), and that other investigators may use the experience gained from our observations and avoid certain sources of error discussed below.

2. Arrangement of observations and sources of error.

The result of extrafocal photometry is liable to be influenced by the following sources of error:

a) variations in the transparency of the terrestrial atmo-

1) They are found, too, in Harvard Annals 71, 2, Standard Region C_5.

2) The only determination of the photographic brightness of Neptune known to the writer is the determination made by K. Schütte in 1920 (Astr. Nachrichten 5130 pp. 357—360).

sphere; b) systematic differences in the sensitiveness of different parts of the photographic plate; c) accidental errors of the measures (or estimations) of density of the photographic image; d) accidental variations in the sensitiveness of the plate near a given point; e) inequality of the distance from the focus for different images; f) influence of the background of the sky and overlapping images of the brighter stars; g) influence of the background (veil) of the plate; h) non-homogeneousness of the extrafocal image; i) errors in the plate constants needed to transform density into stellar magnitude.

The relative importance of these sources of error depends upon the instrument and the brightness of the object under investigation; e. g. for bright objects source f) is negligeable, sources e) and h) can be practically eliminated by making the distance from the focus great enough, and source b) can be reduced to a minimum by placing the images to be compared as near as possible on the plate; the short exposure allows of obtaining a great number of images during a narrow space of time, so that the influence of the sources c) and d) is reduced considerably. The satisfactory results obtained for Mars by the writer [1]) are due chiefly to these favourable circumstances; as the most serious factor affecting the extrafocal photometry of bright objects remains only source a), for the comparison stars are from necessity chosen at an appreciable distance from the object under investigation.

In the case of a faint object like Neptune the latter source of error can be reduced practically to zero, for the comparison stars can be chosen at a small distance from the planet and can be photographed at the same time with the latter. But for a short-focused camera like the camera used in the present investigation sources e) and f) cannot be neglected; source e) — because it is necessary to reduce the distance from the focus to a minimum, lest the time of exposure be increased unreasonably (which means at the same time an increased effect of the background of the sky); and source f) puts a limit to the number of neighbouring images that may be obtained on the same plate. Besides, the effect of the general background of the sky for Neptune proved to be small for the exposures

1) Loc. cit.

(about 10^m) and aperture (F:D = 10) used, so that 6 or more consecutive images could be obtained upon the same plate without impairing the negative; but the effect of overlapping images of stars proved to be a most troublesome source of error, stars of the 10th and 11th magnitude affecting the measured brightness of Neptune in a sensible degree.

Bearing in mind the possible sources of error, the arrangement of the observations was made as follows.

The photographs were obtained with the aid of the 160 mm Petzval camera (focal distance 79,2 cm [1]), scale of focal images 1 mm = 260″.5), at a distance 5—3 mm behind the focus; the latter distance proved to be more convenient, giving fairly uniform extrafocal images measurable on a microphotometer, and with an exposure of 10 minutes producing images of Neptune dense enough for precise determination of its brightness. To control the distance from the focus, an arrangement was adopted analogous to the method of Hartmann for testing objectives. A diaphragm was placed before the objective, having a central aperture of 80 mm and four side-holes, denoted on fig. 1 by a_1, a_2, b_1, b_2; the directions a_1, a_2 and b_1, b_2 were perpendicular, the distances of the centers $a_1\,a_2 = 129{,}9$ mm and $b_1\,b_2 = 130{,}0$ mm; $a_1\,a_2$ was approximatively parallel to the declination circle; the diameters of the side-holes were equal to 20 mm. Thus each extrafocal image of a star reproduced the figure of the diaphragm, having a central disk C and four „satellites“ a_1, a_2, b_1, b_2, as represented on fig. 1. The distance between the „satellites“, measured on a Repsold machine, determines the distance from the focus and, consequently, the intensity of illumination which acted upon the central image C. Since the reduced aperture does not affect the intensity of an extrafocal image, the latter depending solely upon the ratio $\frac{\Delta f}{F}$, where Δf is the distance of the plate from the focus, F — the focal length of the objective, the effect of

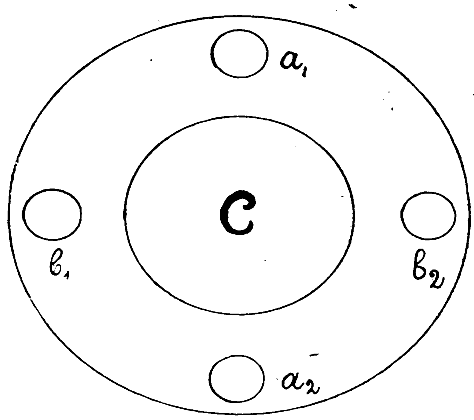

Fig 1.

1) Not 78 cm, as given in Tome XXIV, № 1 of these Publications.

the diaphragm was only an improvement of the photographs, for it reduced the intensity of the background of the sky and the probability of overlapping stellar images about four times. The diameter of the central image *C* was about 0,30—0,35 mm on the majority of the plates.

The photographs were obtained with the aid of the rotating plate-holder, which was constructed by Mr Messer specially for the eclipse expedition of 1914; a description of this plate-holder will be given elsewhere. Here it will be sufficient to say that this arrangement allowed to obtain on a 9 × 12 cm plate two independent photographs of a circular area of 52 mm diameter: a circular window of this diameter, with its centre on the optical axis, was placed before the plate; the latter could be rotated about an axis 107 mm distant from the optical axis and placed in two positions, so that the images of the window fell upon two different places on the plate. The opportunity of obtaining a double number of independent exposures on the same plate was of no little importance, for it allowed of a more precise determination of the plate constants and reduced in this manner the errors originating from the source i). The two positions were denoted as position I and II respectively.

The 200 mm Zeiss refractor was used as guiding telescope [1]; the star (Neptune in all cases except negative № 3 and 4) was placed on the intersection of a horizontal and a vertical thread of the micrometer; 3 vertical and 2 horizontal threads gave six intersection points; their denotations and relative coordinates in minutes of arc are given below:

Intersection Point	Coordinates $\Delta\alpha$	$\Delta\delta$
1	$-0'.2$	$+3'.1$
2	0.0	0.0
3	$+4.7$	$+0.2$
4	$+4.5$	$+3.3$
5	$+9.2$	$+3.6$
6	$+9.4$	$+0.5$

1) The description of the new Zeiss refractor with the photographic camera is given in Tome XXIV № 1 of the Dorpat Publications, 1914. During the war the refractor was dismounted and taken away to Russia, and the observatory succeeded to get it back only in 1921.

As a rule, 3—6 exposures were obtained for the same position using different points of the micrometer network, and the single exposures were denoted by the number of the plate, the position and the number of the intersection-point; e. g. 6 II_4 denotes the exposure on plate №6, position II, intersection-point of the guiding telescope № 4.

Several exposures were obtained with a background illuminated by the moon; in these cases only one exposure in each position was made.

The plates used were Agfa Extra-Rapides, Emulsion 7813, ordinary glass, and in two cases (plates 3 and 4) Agfa-Isolar, Em. 7137; plate 4 was not measured at all; as to plate 3, any systematical difference in the colour sensitiveness between this plate and the bulk of all remaining is not probable, and, moreover, since the plate № 3 enters in our reductions with a small weight, we resolved not to reject altogether this plate.

The plates were developed in a solution of Methol-Hydrochinon during 5 minutes, at a temperature of about 15° C. Before the development they were soaked in distilled water for 10 minutes.

It was impossible to obtain fresh plates here during the observations; a noticeable veil appears on all photographs; if satisfactory results are, nevertheless, obtained, it may be explained partly by the method of reduction: instead of the absolute density the difference between the density of the stellar image and the background was used; with fresh plates even much better results may be expected. On the contrary, the use of plate-glass plates will probably give no sensible improvement, since the method described above eliminates the effect of inequal distance from the focus, produced by the irregular surface of an ordinary plate.

Table 1 contains the data for all exposures of Neptune obtained during spring 1922.

These 13 plates, containing 23 independent photographs with 65 exposures, may be classified as follows: 1) plates or photographs entirely rejected (not measured); such are 3 $I_{(1.2)}$, 4 $II_{(1,2)}$, 15 I, II; the reason for rejecting was the extreme weakness of the images, which rendered any accuracy of the measurements impossible; Neg. 14 must be listed to the same cathegory; it was measured with the chief purpose to obtain data for the

Table 1.

Plate, Pos., Guiding Point	Date, Sidereal Time (Middle of Exposure)	Exposure Seconds	Guiding Star	Plate Sort	△f mm	General Remarks
	1922					
3 I 1	March 18, 9h.2	722.5	BD 16°1901	Agfa-Isolar	5.1	Very weak
2	„ 9.5	280.0	17°2018			
3 II 6	March 20, 9.1	722.5	16°1901		4.9	weak negative
5	9.3	725.0	17°2007			
4	9.7	731.0	17°2018			
3	10.0	720.5	16°1901			
2	10.4	720.5	„			
1	10.7	730.0	„			
(7)	11.0	210.0	„			
4 II 1	March 20, 12.5	721.5	BD 16°1901	Agfa-Isol.	4.9	very weak
2	12.8	720.5	„			
5 I 1	March 21, 8.4	600.5	Neptune	Agfa-Extra-Rapid.	5.3	weak negative
2	8.7	480.5	„			
3	9.0	723.0	„			
4	9.3	595.0	„			
5	9.7	602.0	„			
6	10.3	610.5	„			
5 II 1	10.7	600.0	„		4.9	
2	11.0	599.5	„			
3	11.2	601.0	„			
4	11.5	590.5	„			
5	11.8	602.0	„			
6	12.1	609.0	„			
6 I 1	March 22, 8.5	606.0	„	„	3.4	good
2	9.0	600.5	„			
3	9.2	603.0	„			
4	9.5	602.0	„			
5	9.8	1200.0	„			
6	10.2	599.5	„			
6 II 1	10.6	599.0	„		3.2	
2	10.8	388.0	„			
3	11.1	594.0	„			
4	11.5	599.5	„			
5	11.7	600.5	„			
6	12.0	600.5	„			
7 I 1	March 23, 8.6	600.0	„		3.4	
2	8.8	650.0	„			

Table 1. Continued.

Plate Pos., Guiding Point	Date, Sidereal Time	Exposure Seconds	Guiding Star	Plate Sort	$\triangle f$ mm	General Remarks
	1922					
7 I 5	March 23, 11h.0	600.5	Neptune	Agfa Extra-Rap. 7813		
6	11.3	601.0			3.4	
3	11.6	630.0				
4	11.8	599.5				good
7 II 1	12.1	601.5				
2	12.4	600.5			3.0	
3	12.7	600.5				
8 I 2	March 24, 8.8	600.5	„	„		
4	9.1	600.5				
6	9.3	600.0			3.5	good; after the end of the exposures clouds.
8 II 1	9.6	600.0				
4	9.9	600.0				
5	10.1	600.0				
10 I 3	April 8, 11.5	455.0	„	„	3.4	After the end of the exposure clouds. Weak negative; moonlit background
11 I 3	April 12, 10.8	600.0	„	„	3.5	
II 3	„ 11.0	601.5			3.1	
12 II 3	„ 11.4	600.0			3.3	moonlit background (1 day after full moon); weak negatives
I 3	„ 11.6	600.5			3.5	
13 I 3	„ 11.9	600.5			3.4	
II 3	„ 12.4	599.5			3.1	
14 I 1	April 17, 11.6	599.5	„	„		
4	„ 11.8	600.5			3.5	
II 4	„ 12.1	600.0				strong atmospherical absorption; very weak negatives.
3	„ 12.3	599.5				
15 II 3	„ 12.7	600.5			3.5	
I 3	„ 12.9	600.0				
16 I 1	April, 23, 12.5	759.5	„	„	3.6	
4	12.8	759.5				

derivation of the relation between density and magnitude at the lowest values of the density; 2) weak negatives: 3 II; 5 I. II; 10, 11, 12, 13, 16; the density of the images upon these photographs being great enough to allow of a determination of the mean brightness of the comparison stars, but too low for the derivation of the individual values of the brightness of Neptune (probable error of one exposure $\pm$ 0.08 st. mg.); the faintness of the images upon these negatives has two causes: for plates 3 and 5 — the relatively short exposure for the somewhat great distance from the focus; for plates 10, 11, 12, 13 — the intense moonlit background (near full moon); for pl. 16, as well as for 14 and 15, — an abnormally large atmospherical absorption, which reduced the photographic brightness by about 1 magnitude; 3) good negatives, of normal density; these are: pl. 6 I, II; 7 I, II; 8 I, II; their probable error of one determination of the brightness of Neptune did not surpass $\pm 0^{mg}.03$, as will be shown later on; for the purpose of testing the variability of Neptune only these plates could be used; they contain 26 measured exposures of the planet (exposure 7 I 6 was rejected because of the superposition of the images of Neptune and the comparison-star B. D. 17°2018). A favourable circumstance is that all these exposures refer to 3 consecutive nights.

Table 2 contains the list of the comparison stars used together with their rectilinear coordinates x and y on the photographs in mm, the y-axis being taken towards the north pole, and the position of Neptune March 24 at transit at Greenwich (it nearly coïncides with the middle of the observations at Dorpat) being taken as the origin.

Table 2.

Comparison Star	B. D.	mg.	x (mm)	y (mm)
a	16°1901	7.5	— 6.0	— 10.3
b	17 2007	7.5	— 6.0	+ 7.0
b'	17 2004	8.0	— 7.7	+ 10.8
c	17 2018	8.0	+ 1.7	+ 1.4
k	15 1981	8.1	— 4.1	— 19.2
g	17 2032	7.7	+ 15.3	— 0.9

The denotations of the comparison stars given in the first column of this table will be used further.

The stars *a* and *b* are contained in Harvard Annals vol. 71, 2, p. 35 (Standard Region C_5) and in the Göttingen Actinometrie, Teil B, p. 59, as well as in the Potsdam Durchmusterung, whence the following data are taken:

Star	Spectrum Harvard	Photogr. Magnitude		Visual Magn.	
		Harvard	Göttingen Actinom.	Harvard	Potsdam
a	F_5	7.78	7.22	7.27	7.59
b	K	8.74	8.33	7.57	7.58

The difference in the photographic magnitude of these stars is: according to Harvard $b - a = 0.96$ st. mg.
„ Göttingen Actinometrie $b - a = 1.11$ „

A rough determination with the aid of the Petzval camera, based on an approximate value of Schwarzschild's exponent p, gave $b - a = 1{,}2 \pm 0{,}1$ st. mg.; taking into account that the Harvard values are not direct determinations, but were derived from the visual magnitudes by applying mean colour-indices, and that the value of the Göttingen Actinometrie is much more near to our own estimate, we shall adopt further as the basis of our magnitude scale according to Schwarzschild:

$$b - a = +1{,}11 \text{ st. mg.}$$

The magnitude of the comparison stars on an absolute scale is of no importance in our present investigation; we shall assume $a = 0.00$ and $b = +1.11$; the magnitudes of the remaining 4 comparison stars will be derived from the photographs by interpolation between b and a, and the magnitude of Neptune will be determined relatively to the mean magnitude of the bulk of the comparison stars, their greater number reducing the accidental error of one individual determination of the brightness and serving at the same time as a guarantee against occasional variability.

3. Measurement of the plates.

The density of the photographs was measured on a microphotometer of Hartmann, constructed by Mr. Messer at Dorpat (the prism of Lummer-Brodhun being from C. Zeiss); the area measured was a circle of a diameter corresponding to 0.113 mm on the plate. The circle was always placed on the centre of the extrafocal image, and the negative as well as the wedge were observed a little out of focus to make the grains of silver disappear; the degree of diffuseness thus produced was about 0.03 mm, so that no trouble could arise in measuring surfaces of 0.3 mm diameter and more. A number of measures of the background near the stellar image was taken, chiefly in the two opposite sectors b_1 a_1 or a_2 b_2 (Fig. 1), the number of these measures being equal to the number of the measures of the central image; but whereas the latter measures referred always to the same point — the centre of the image C —, every measure of the background was made on a new point, the points being equally distributed between the two sectors, but within the limits of the latter chosen by chance. The distance of the points of the background from the edge of the central image C was about 0,15—0,20 mm.

The number of readings on the microphotomer was as a rule: for Neptune 4, for the comparison-stars 2; if the measures differed by more than 0,10—0,13 units of the scale (for normal exposures the unit was about 1 st. mg.), a double number of readings was taken; in computing the mean value the discordant measures were not rejected. For each pair of measures the two readings were obtained in moving the scale in two opposite directions.

The distance between the side-images b_1 b_2 and a_1 a_2 (fig. 1) was measured with the x-micrometer of the Repsold machine; to save time the micrometer-screw was used instead of the scale; errors from the inclination of the plate, affecting the scale of the image within the field of the microscope, proved to be negligeable for the distances measured (0.5—0.8 mm); the unit is one revolution of the micrometer-screw, practically equal to 1 mm. To obtain both distances (b_1 b_2 and a_1 a_2) with the same micrometer, the plate was placed consecutively into two positions, differing by 90°. Only one measure of each diameter of a given image was taken, the accuracy of these measures surpassing the accuracy required for photometric purposes.

Table 3 gives a sample of the measures; the comparison stars are denoted by the letters given in the first column of table 2; Neptune is denoted by n; the guiding point is indicated by the number accompanying the letter of the star.

4. Effect of the varying distance from the focus.

As a measure of the distance from the focus we will take the mean of the measured distances a_1 a_2 and b_1 b_2, and call it

Table 3.
Negative 8, I.

Star and Image	Microphotometer Readings: Star	Mean	Background	Mean	Distances mm: b_1 b_2	a_1 a_2
n_2	4.59 4.58 4.58 4.58	4.58	3.72 3.74 3.68 3.72	3.72	0.574*)	0.585*)
n_4	4.63 4.52 4.59 4.60	4.59	3.73 3.66 3.70 3.77	3.72	.568*)	.579*)
n_6	4.68 4.62 4.70 4.63	4.66	3.76 3.65 3.73 3.74	3.72	.568*)	.574*)
Mean				3.72	0.577	
c_2	4.23 4.18	4.21	3.70 3.66	3.68	.565	.593
c_4	4.34 4.34	4.34	3.73 3.65	3.69	.575	.578
c_6	4.38 4.36	4.37	3.70 3.72	3.71	.572	.586
Mean				3.69	0.578	
b_2	4.36 4.25 4.22 4.29	4.28	3.78 3.74	3.76	.584	—**)
b_4	4.29 4.27	4.28	3.67 3.67	3.67	.572	.564
b_6	4.30 4.25	4.28	3.69 3.71	3.70	.588	.565
Mean				3.71	0.575	
b'_2	4.69 4.67	4.68	3.67 3.66	3.67	.577	.584
b'_4	4.83 4.80	4.82	3.73 3.70	3.72	.580	.581
b'_6	4.83 4.82	4.83	3.74 3.70	3.72	.571	.567
Mean				3.70	0.575	
a_2	5.54 5.51	5.53	3.54 3.66	3.60	.569	.558
a_4	5.54 5.56	5.55	3.59 3.57	3.58	.558	.559
a_6	5.65 5.65	5.65	3.73 3.65	3.69	.553	.553
Mean				3.62	0.559	
g_2	5.14 5.14	5.14	3.74 3.77	3.76	—	.582
g_4	5.17 5.23	5.20	3.74 3.77	3.76	—	.584
g_6	5.28 5.26	5.27	3.75 3.78	3.77	—	.587
Mean				3.76		0.584
k_2	4.64 4.66	4.65	3.60 3.66	3.63	.561	—
k_4	4.70 4.76	4.73	3.62 3.64	3.63	.562	—
k_6	4.76 4.76	4.76	3.69 3.67	3.68	.560	—
Mean				3.65	0.561	

*) Mean of two measures.
**) Images defective.

simply the **d i a m e t e r** (d) of the image[1]); for an objective without spherical aberration the intensity of illumination of an extrafocal image varies as $\frac{1}{d^2}$; in the actual case many sources of error affecting the diameter or the intensity arise, but for small differences the following differential formula may be safely applied:

$$\triangle i = -\frac{2\triangle d}{d} = -\frac{2\triangle f}{f} \quad (1) \text{ or}$$

$$\triangle m = +\frac{2{,}1\triangle d}{d} = +\frac{2{,}1\triangle f}{f} \quad (2),$$

where $\triangle i$ and $\triangle m$ are the **d e v i a t i o n s** of the brightness and stellar magnitude respectively, produced by a variation of the diameter or the focal distance equal to $\triangle d$ or $\triangle f$ respectively (f is the distance from the focus).

The Petzval objective was investigated according to the method of Hartmann by Mr. V. Berg, formerly assistant of the observatory; his unpublished results for the spherical aberration are contained in table 4; r denotes here the radius of the zone, $\triangle f$ — the deviation of the focus from the focus of the zone $r = 65$ mm (corresponding to our side-images b_1 b_2, a_1 a_2);

A positive deviation corresponds to a greater distance from the objective. As may be inferred from the table, the spherical aberration is small enough to allow of the application of the differential formulae (1) or (2), f being of the order of 3—5 mm.

T a b l e 4.

r mm	$\triangle f$ mm	r mm	$\triangle f$ mm
7	— 0.17	47	— 0.04
12	+ 0.06	52	— 0.06
17	+ 0.03	57	— 0.05
22	+ 0.05	60	— 0.01
27	+ 0.03	62	— 0.03
32	+ 0.03	67	+ 0.03
37	+ 0.01	72	+ 0.01
42	— 0.01	75	+ 0.03
45	— 0.01		

The astigmatism of the objective is small too; according to the investigation of V. Berg, its dependence on the position-angle may be represented by a sinusoide with an amplitude of 0.02 mm.

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1) Multiplying the diameter by 6,1 we obtain the true distance from the focus.

A supposed interaction (attraction or repulsion) of the photographic images must be too small to affect formulae (1) or (2); a photographic test made specially for this purpose indicated a repulsion of about 0.005 mm for a diameter 0.66 mm; for a diameter of 0,50 mm the effect must thus be smaller than 0,01 mm.

A more serious danger arose from the position of the diaphragm: this was placed 35 cm in front of the objective on the protecting-tube, the arrangement for attaching the diaphragm directly on the objective being not ready at the time of observation. The consequence was that the light from different comparison stars passing the side-holes did not meet exactly the same points on the objective; but 1) since the optical quality of the objective is uniform enough and 2) since the comparison stars preserved during the period of observation approximately the same position with respect to the centre of the field, i. e. Neptune, — the inconvenient position of the diaphragm caused but little trouble[1]); the only consequence was that for the more distant comparison stars g and k one diameter could not be measured, one of the corresponding side-holes falling partly without the objective; the diameter not measured was: for g — $b_1\ b_2$, and for k — $a_1\ a_2$ (compare table 3).

Systematical differences between the diameters $b_1\ b_2$ and $a_1\ a_2$ occurred; these differences as determined from plates 6, 7, 8, 10, 11, 12, 13, are given in table 5. The 3[d] line of this

Table 5.

Star	n	c	a	b	b'	Mean
Syst. Difference $b_1\ b_2 - a_1\ a_2$ mm	− 0.007	− 0.007	+ 0.003	+ 0.003	+ 0.008	0.000
P.E. of 1 Distance	± 0.0045	0.0051	0.0030	0.0075	0.0046	

table contains the probable error of 1 measure of a distance $a_1\ a_2$ or $b_1\ b_2$; the accuracy of the diameter depends upon the brightness: for the fainter stars less accurate measures were obtained. This probable error of one measure of the diameter

1) It may be easily conceived that for differential measures only the constancy, not the equality of the focus of the side-holes is required.

corresponds to the following probable error in the photometric magnitude, computed for $d = 0.5$ mm and 0.8 mm respectively:

Table 6.

P. E. of 1 Diameter. St. mg.	n	c	a	b	b'	g, k	Average
($d = 0.500$ mm)	$\pm$0.018	0.020	0.012	0.030	0.019	(0.020)	$\pm$0.020
($d = 0.800$ mm)	$\pm$0.012	0.013	0.008	0.019	0.012	(0.013)	

For a group of neighbouring images of the same star the average diameter was computed; since one image gives usually 2 measures (a_1 a_2 and b_1 b_2), the accuracy of the average diameter of a group containing 3—6 images is great in comparison with the photometric estimations. In cases when only one distance of a certain image was measured[1]), a correction equal to one half of the systematical difference given in table 5 was applied; for the stars g and k the measures of the single distance were adopted without any systematical correction. In this way the mean diameters of tables 3 and 7 were computed.

5. Influence of the background.

The microphotometer readings can be used for interpolation of the brightness in the following three ways: 1) using the density of the stellar image directly, without taking into consideration the background; 2) taking the difference of density between the stellar image and the background immediately surrounding, or the so-called individual background; 3) taking the difference of density between the stellar image and the average background surrounding the entire group of neighbouring images.

The choice between these alternatives could be made only on the basis of the observational data.

For the 3 best negatives (№ 6, 7 and 8) the magnitudes of the stars b', c, g and k were provisionally determined by a linear interpolation of the microphotometer readings, the stars a and b being taken as the basis.

1) The other distance being rejected because of the defectiveness of the images.

The correction for diameter according to formula (2) was applied; the single measures were combined in groups according to the two positions on the plate, and the 6 groups (6 I. II; 7 I. II; 8 I. II) for each of the 4 stars gave together 24 magnitude-determinations, whence the following values of the probable error of one group (containing on the average 4 images[1])) were found:

Using the directly measured density .... p. e. = ± 0,045 st. mg.
" " difference of density between
image and average background .. p. e. = ± 0,031 st. mg.

The advantage of taking into account the background is obvious. The relative magnitudes of the comparison stars found thus as a first approximation were:

Star	*a*	*b*	*b′*	*c*	*g*	*k*
Magnitude	0.00	1.11	0.70	1.16	0.38	0.75.

These magnitudes may be compared with the definitive magnitudes given later on.

The following question must be answered next: in taking the difference of photographic density, which background is to be preferred — the individual or the average one? To answer this question, the differences $\triangle$ and $\triangle_0$ were formed for 60 groups of the 7 stars (including Neptune) with the number of images $\geqslant 3$ on the negatives 3 II, 5 I. II, 6 I. II, 7 I. II, 8 I. II, $\triangle$ denoting the difference between the stellar image and the individual background, $\triangle_0$ — the difference with respect to the average background.

The differences between the extreme values within each group were tabulated; the following result was obtained:

Mean square of the difference of the extreme values

of $\triangle$ . . . ± 0,115 }
of $\triangle_0$ . . . ± 0.097 } Units of the microphotometer scale.

These data are decidedly in favour of the average background. Thus further we shall use only the differences $\triangle_0$ as argument of interpolation.

6. Result of the measurement of the plates.

Table 7 contains the result of all measures of the plates. The first column contains the denotation of the star and of the

1) Only exposures near 600^s were used.

Table 7.

Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$
				Negative 3, II							
n_1	0.28	0.801 (9)	0.	c_1	0.16		—0.002	b_1	0.23		+0.051
n_2	0.25		»	c_2	0.13		»	b_2	0.21		»
n_3	0.31		»	c_3	—			b_3	0.15		.050
n_4	0.19		»	c_4	0.21	—	—0.001	b_4	0.16	—	.051
n_5	0.25		»	c_5	0.17		»	b_5	0.13		»
n_6	0.19		»	c_6	—			b_6	0.20		»
n_7	0.18	—	0.	c_7	0.16		—0.002	b_7	—	—	
b_1'	0.43	0.821 (7)	+0.048	a_1	0.87	0.797 (9)	0.000	g_1	0.58	0.797 (3)	—0.015
b_2'	0.38		»	a_2	0.98		»	g_2	0.61		—0.013
b_3'	0.36		»	a_3	0.95		—0.001	g_3	0.64		—0.013
b_4'	0.35		»	a_4	1.00		—0.002	g_4	0.70		—0.012
b_5'	0.32		»	a_5	0.99		—0.003	g_5	0.63		—0.011
b_6'	0.28		»	a_6	0.83		»	g_6	0.55		—0.010
b_7'	0.25		+0.049	a_7	0.71	0,546	0.000	g_7	0.39	—	—0.016
k_1	0.39	0.782 (4)	—0.035	k_3	0.37	0.782	—0.038	k_5	—	0.782	
k_2	0.26		—0.036	k_4	0.37		»	k_6	0.31		—0.039
								k_7	—		
				Negative 5, I							
n_1	0.26	0.830 (7)	0.	c_1	0.09	0.841 (1)	+0.028	b_1	0.15	0.845 (1)	+0.034
n_2	0.22		»	c_2	0.07		»	b_2	0.13		»
n_3	0.34		»	c_3	0.11		»	b_3	0.18		»
n_4	0.28		»	c_4	0.16		»	b_4	0.15		»
n_5	0.18		»	c_5	0.13		»	b_5	0.15		»
n_6	0.17		0.	c_6	0.10		+0.027	b_6	0.14		+0.035
b_1'	0.27	0.838 (8)	+0,012	a_1	0.75	0.827 (9)	+0.001	g_1	0.46	0.819 (4)	—0.027
b_2'	0.27		.012	a_2	0.65		»	g_2	0.39		.028
b_3'	0.36		.013	a_3	0.91		»	g_3	0·59		.029
b_4'	0.32		.014	a_4	0.74		»	g_4	0.52		.030
b_5'	0.27		.014	a_5	0.83		»	g_5	0.53		.031
b_6'	0.27		+0.015	a_6	0.80		+0.002	g_6	0.52		—0.032
k_1	0.29	0.830 (5)	+0.015	k_3	—	0.830	+0.015	k_5	0.31	0.830	+0.015
k_2	—		»	k_4	0.33		»	k_6	0.27		»
				Negative 5, II							
n_1	0.31	0.808 (8)	0.	c_1	0.19	0.808 (4)	—0.002	b_1	0.16	0.811 (7)	+0.002
n_2	0.30		»	c_2	0.17		—0.002	b_2	0.26		»
n_3	0.26		»	c_3	0.14		—0.003	b_3	0.23		+0.003
n_4	0.26		»	c_4	—		»	b_4	0.25		»
n_5	0.31		»	c_5	0.15		—0.004	b_5	0.12		+0.002
n_6	0.34		»	c_6	0.14		—0.005	b_6	0.13		»

Table 7. Continued.

Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$
				Negative 5, II							
b_1'	0.37		+0.003	a_1	1.00		—0.024	g_1	0.59		—0.041
b_2'	0.34		+0.004	a_2	0.96		„	g_2	0.56		—0.042
b_3'	0.42	0.812	„	a_3	0.91		—0.023	g_3	0.62	0.794	—0.044
b_4'	0.32	(8)	„	a_4	0.95	0.795	—0.022	g_4	0.57	(6)	—0.045
b_5'	0.31		+0.003	a_5	0.90	(12)	—0.021	g_5	0.63		—0.045
b_6'	0.35		+0.002	a_6	0.91		—0.019	g_6	0.58		—0.047
k_1	0.31	0.795	—0.016	k_3	0.35	0.795	—0.016	k_5	0.41	0.795	—0.013
k_2	0.37	(6)	„	k_4	0.39		—0.014	k_6	0.34		—0.010
				Negative 6, I							
n_1	0.74		0.	c_1	0.40		+0.003	b_1	0.56		+0.006
n_2	0.73		„	c_2	0.45		„	b_2	0.61		„
n_3	0.74	0.554	„	c_3	0·49	0.555	„	b_3	0.57	0.557	„
n_4	0.81	(12)	„	c_4	0.46	(12)	„	b_4	0.55	(10)	„
n_5	1.36		„	c_5	0.89		„	b_5	1.03		„
n_6	0.79		„	c_6	0.46		+0.002	b_6	—		
b_1'	0.90		—0.022	a_1	1.70		—0.027	g_1	1.20		—0.009
b_2'	0.90		—0.021	a_2	1.76		„	g_2	1.37		—0.011
b_3'	0.94	0.551	—0.021	a_3	1.72	0.545	„	g_3	1.23	0.551	—0.012
b_4'	1.01	(12)	—0.020	a_4	1.75	(12)	„	g_4	1.29	(6)	—0.012
b_5'	1.51		—0.020	a_5	2.14		„	g_5	1.75		—0.013
b_6'	1.01		—0.019	a_6	1.70		—0.026	g_6	1.30		—0.014
k_1	0.83	0.554	+0.015	k_3	0.93	0.554	+0.015	k_5	1.47	0.554	+0.015
k_2	—	(5)	„	k_4	0.91		„	k_6	0.91		„
				Negative 6, II							
n_1	1.06		0.	c_1	0.63		—0.019	b_1	0.67		+0.024
n_2	0.78		„	c_2	0.46		„	b_2	0.45		+0.023
n_3	1.00	0.523	„	c_3	0.63	0.519	„	b_3	0.66	0.530	„
n_4	1.01	(12)	„	c_4	0.61	(12)	—0.020	b_4	0.67	(11)	+0.024
n_5	0.93		„	c_5	0.59		—0.021	b_5	0.66		+0.023
n_6	0.92		„	c_6	0.51		„	b_6	0.64		+0.022
b_1'	1.12		+0.017	a_1	1.96		—0.058	g_1	1.55		—0.068
b_2'	0.75		+0.018	a_2	1.65		„	g_2	1.19		—0.069
b_3'	1.10	0.529	„	a_3	1.96	0.509	„	g_3	1.57	0.510	—0.070
b_4'	1.17	(12)	„	a_4	1.99	(11)	—0.056	g_4	1.56	(6)	—0.072
b_5'	1.10		+0.017	a_5	1.87		—0.054	g_5	1.48		—0.073
b_6'	1.11		+0.016	a_6	1.94		—0.053	g_6	1.49		—0.073
k_1	1.12	0.507	—0.059	k_3	1.21	0.507	—0.058	k_5	1.19	0.507	—0.054
k_2	0.89	(3)	—0.058	k_4	1.12		—0.056	k_6	1.11		—0.052

Table 7. Continued.

Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$
				Negative 7, I							
n_1	0.68		0.	c_1	—			b_1	0.47		−0.021
n_2	0.80		„	c_2	0.48		−0.001	b_2	0.54		„
n_3	0.84	0.559	„	c_3	0.46	0.559	−0.003	b_3	0.56	0.555	„
n_4	0.77	(12)	„	c_4	—	(11)		b_4	0.49	(12)	−0.022
n_5	0.71		„	c_5	0.44		−0.002	b_5	0.50		„
n_6	—			c_6	0.42		−0.003	b_6	0.49		−0.021
b_1'	0.90		−0.040	a_1	1.49		+0.008	g_1	1.10		−0.017
b_2'	0.91		„	a_2	1.69		„	g_2	1.19		−0.018
b_3'	0.91	0.551	−0.038	a_3	1.59	0.559	+0.013	g_3	1.24	0.554	−0.027
b_4'	0.84	(12)	−0.039	a_4	1.55	(12)	+0.015	g_4	1.16	(6)	−0.027
b_5'	0.91		−0.038	a_5	1.59		+0.011	g_5	1.20		−0.025
b_6'	0.93		„	a_6	1.59		+0.013	g_6	1.23		−0.026
k_1	0.81	0.576	+0.080	k_3	0.83	0.576	+0.087	k_5	0.80	0.576	+0.085
k_2	0.93	(5)	+0.080	k_4	0.82		+0.089	k_6	0.75		+0.087
				Negative 7, II							
n_1	1.02	0.491	0.	c_1	0.67	0.493	+0.004	b_1	0.66	0.499	+0.030
n_2	0.95	(6)	„	c_2	0.59	(6)	„	b_2	0.67	(6)	+0.029
n_3	0.92		„	c_3	0.55		+0.003	b_3	0.59		„
b_1'	—			a_1	1.79	0.488	+0.003	g_1	1.55	0.475	−0.081
b_2'	1.08	0.492	−0.003	a_2	1.71	(6)	+0.004	g_2	1.54	(3)	−0.083
b_3'	1.05	(6)	−0.005	a_3	1.79		+0.008	g_3	1.53		−0.085
k_1	1.01	0.497 (3)	+0.053	k_2	1.06	0.497	+0.056	k_3	0.99	0.497	+0.060
				Negative 8, I							
n_2	0.86	0.577	0.	c_2	0.52	0.578	+0.003	b_2	0.57	0.575	−0.013
n_4	0.87	(6)	„	c_4	0.65	(6)	„	b_4	0.57	(5)	„
n_6	0.94		„	c_6	0.68		„	b_6	0.57		„
b_2'	0.98	0.575	−0.016	a_2	1.91	0.559	−0.060	g_2	1.38	0.584	+0.028
b_4'	1.12	(6)	„	a_4	1.93	(6)	„	g_4	1.44	(3)	„
b_6'	1.13		„	a_6	2.03		„	g_6	1.51		„
k_2	1.00	0.561 (3)	−0.047	k_4	1.08	0.561	−0.047	k_6	1.11	0.561	−0.047
				Negative 8, II							
n_1	1.04	0.569	0.	c_1	0.66	0.573	+0.014	b_1	0.67	0.573	+0.009
n_4	1.07	(6)	„	c_4	0.60	(6)	+0.015	b_4	0.72	(5)	„
n_5	1.01		„	c_5	0.62		+0.014	b_6	0.69		+0.010

Table 7. Continued.

Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$
				Negative 8, II							
b_1'	1.17	0.577	+0.022	a_1	2.15	0.555	—0.044	g_1	1.55	0.563	—0.024
b_4'	1.18	(6)	„	a_4	2.02	(6)	—0.044	g_4	1.50	(3)	„
b_5'	1.14		+0.023	a_5	2.00		—0.043	g_5	1.50		—0.025
k_1	1.17	0.554 (3)	—0.042	k_4	1.12	0.554	—0.042	k_5	1.16	0.554	—0.042
Negative 10				**Negative 11, I**				**Negative 11, II**			
n	0.46	0.567 (2)	0.	n	0.45	0.578 (2)	0.	n	0.66	0.513 (2)	0.
c	0.32	0.554 (2)	—0.052	c	0.29	0.565 (2)	—0.052	c	0.49	0.509 (2)	—0.019
b	0.34	0.564 (2)	—0.016	b	0.34	0.584 (2)	+0.018	b	0.49	0.524 (2)	+0.041
b'	0.66	0.557 (2)	—0.045	b'	0.47	0.573 (2)	—0.027	b'	0.77	0.506 (2)	—0.037
a	1.00	0.548 (2)	—0.059	a	1.08	0.572 (2)	—0.012	a	1.28	0.505 (2)	—0.024
g	0.77	0.577 (1)	+0.030	g	0.72	0.582 (1)	+0.011	g	1.04	0.516 (1)	+0.008
k	0.56	0.561 (1)	—0.002	k	0.55	0.578 (1)	+0.018	k	0.70	0.524 (1)	+0.066
Negative 12, I				**Negative 12, II**				**Negative 13, I**			
n	0.52	0.565 (2)	0.	n	0.58	0.551 (2)	0.	n	0.58	0.557 (2)	0.
c	0.34	0.572 (2)	+0.023	c	0.35	0.535 (2)	—0.067	c	0.35	0.566 (2)	+0.032
b	0.37	0.567 (2)	+0.001	b	0.38	0.544 (2)	—0.033	b	0.34	0.567 (2)	+0.033
b'	0.57	0.566 (2)	—0.006	b'	0.57	0.544 (2)	—0.035	b'	0.52	0.556 (2)	—0.012
a	1.04	0.562 (2)	+0.003	a	1.12	0.527 (2)	—0.083	a	1.17	0.553 (2)	0.000
g	0.78	0.576 (1)	+0.035	g	0.89	0.534 (1)	—0.074	g	0.84	0.554 (1)	—0.020
k	0.50	0.584 (1)	+0.097	k	0.54	0.534 (1)	—0.047	k	0.66	0.589 (1)	+0.148
Negative 13, II				**Negative 14, I**							
n	0.60	0.520 (2)	0.	n_1	0.15	—	0.	b_4'	0.22	—	—0.017
c	0.37	0.507 (2)	—0.060	n_4	0.18			a_1	0.53	0.568	—0.012
b	0.41	0.517 (2)	—0.020	c_1	0.07	—	(-0.010)	a_4	0.46	(3)	—0.010
b'	0.54	0.519 (2)	—0.013	c_4	0.06			g_1	0.40	0.562	—0.038
a	1.18	0.508 (2)	—0.031	b_1	0.13	—	(0.000)	g_4	0.40	(1)	—0.040
g	0.88	0.500 (1)	—0.094	b_4	0.17			k_1	0.17	—	(+0.036)
k	0.68	0.518 (1)	+0.021	b_1'	0.22	—	(-0.017)	k_4	0.20		
Negative 14, II				**Negative 16.**							
n_3	0.24	0.540	0.	n_1	0.21	0.605	0.				
n_4	0.22	(4)	0.	n_4	0.29	(2)	0.				
c_3	—			c_1	0.13	—	(-0.010)				
c_4	—			c_4	0.13		(-0.013)				
b_3	0.12	—	(0.000)	b_1	0.21	—	(-0.014)				
b_4	0.14			b_4	0.17		(-0.015)				
b_3'	0.26	0.552	+0.033	b_1'	0.34	0.591	—0.071				
b_4'	0.23	(2)		b_4'	0.32	(4)	—0.073				

NB. The values of the correction $\triangle m$ given in parantheses are found on the assumption of a zero correction for the distance from the focus, the diameter in these cases being not measured; the probable error of such a value of $\triangle m$ is about ± 0.03 st. mg.

Table 7. Continued.

Star, Image	$\triangle_0$	d	$\triangle m$	Star, Image	$\triangle_0$	d	$\triangle m$
Negative 14, II				Negative 16.			
a_3	0.63	0.524 (4)	—0.031	a_1	0.74	0.590 (4)	—0.017
a_4	0.69			a_4	0.82		—0.011
g_3	0.48	0.536 (2)	—0.036	g_1	0.51	0.578 (2)	—0.122
g_4	0.45			g_4	0.57		—0.127
k_3	0.33	—	(+0.045)	k_1	0.31	—	+0.059
k_4	0.30			k_4	0.29		+0.068

exposure; the second column gives the difference $\triangle_0$ of the microphotometer readings between the image of the star and the average background of the corresponding group of images; the third column contains the mean value of the diameter, the number of measured distances being given in parentheses; the fourth column ($\triangle m$) contains the deviation of the brightness from a normal value, expressed in stellar magnitudes; this quantity will be explained later on. For several images the density or the diameter was not measured either because of some defects of the images (superposition etc.) or their extreme faintness (as for Neg. 14, where many diameters were not measured because the side-images were not seen).

The value of $\triangle m$ was obtained in the following way. Let us call the normal intensity of the image of a certain star its intensity when at the same zenithal distance and distance from the focus as Neptune; then $\triangle m$ represents the deviation from this normal value, the effect of the distance from the focus being given by (2), and the differential absorption being given by

$$\triangle m' = \gamma \, (\mathrm{Sec}\, Z_s - \mathrm{Sec}\, Z_n) \quad . \; . \; . \; (3),$$

Z_s and Z_n denoting the zenith distances of the star and Neptune, and γ being the photographic coefficient of absorption expressed in stellar magnitudes. For the days of normal transparency (all days except April 17 and 23) γ was assumed equal to $0^{mg}_{.}48$, which is the value found by the writer for Moscow in 1914 [1]); for the abnormal days of April 17 and 23 (Plates 14 and 16) a double value of γ was assumed, which does not seem to be exaggerated, since the actinic effect was on these days reduced

1) Astr. Nachr. 5162 p. 19; $\gamma = -2{,}5 \log a = +0{,}48$.

by about 1 st. magnitude. Besides, the accurate value of the coefficient of absorption is of no importance, the reductions being small (only for two stars they were greater than $0^{m}_{\cdot}g01$), and the stars being placed symmetrically enough around Neptune.

The $\triangle m$ of table 7 represent deviations; the correction of a magnitude measured on the plate will be then equal to — $\triangle m$.

7. The formula of interpolation and the plate constants.

The differential brightness of Neptune is to be found with the aid of interpolation between stars with a given magnitude interval. We may write generally

$$m_1 - m_2 = f(\triangle_1, \triangle_2) \quad . \; . \; . \quad (4),$$

where m_1 and m_2 are the stellar magnitudes, $\triangle_1$ and $\triangle_2$ — the measured densities of the two stars.

The function (4) will be generally represented by a curve, the density-function of the photometric wedge and of the measured plate being not identical. The form of the curve must be found empirically. As a good first approximation for normal exposures a linear interpolation can be used; and if all our negatives were of uniform density, the form of the interpolation curve would be of no importance for differential observations, the curvature affecting equally all measures. But in our case plates of a different quality were obtained; and it was estimated that a simple linear interpolation would lead to a systematical error of about 0.1 stellar magnitudes for the faintest images of Neptune; thus the curvature of the density-function of the plates could not be neglected. The form of the function $f(\triangle_1, \triangle_2)$ was found in the following way. The exposures were grouped according to the density of the images and mean values of $\triangle_0$ for each star were found; the result is given in table 8; *N* denotes the number of exposures.

The mean $\triangle_0$ were plotted as ordinates with the provisional magnitudes as abscissae (no corrections for varying diameter and absorption were applied); thus seven curves representing the preliminary form of the function $f(\triangle_0)$ for different values of $\triangle_0$ were found; for a given $\triangle_0$ these curves are shifted along the x-axis by a certain amount which can be expressed in stellar magnitudes; the shifts were read from the

Table 8.

Group	Exposures used	N	Mean $\triangle_0$ *c*	*b*	*k*	*b′*	*g*	*a*
A	14 I.II ; 15 I_2	5	0.07	0.14	0.25	0.24	0.43	0.65
B	16 I ; 5 $I_{1.4-6}$	6	0.12	0.16	0.30	0.30	0.52	0.78
C	5 II ; 5 I_3	7	0.15	0.19	0.36	0.35	0.59	0.94
D	10; 11; 12; 13	7	0.36	0.38	0.60	0.58	0.85	1.13
E	7 $I_{1.4-6}$	4	0.43	0.49	0.80	0.90	1.17	1.56
F	6 I_{1-4}.7II_{1-3}	7	0.54	0.61	0.96	1.01	1.42	1.75
G	6 $II_{1-3.6}$ 8 I.II	11	0.61	0.65	1.19	1.12	1.51	1.98
Provisional Magnitudes			1.16	1.11	0.75	0.70	0.38	0.00

curves for the points corresponding to the star *a* (these points being more trustworthy as having the greatest $\triangle_0$), and were found as follows:

Groups	*A—B*	*B—C*	*C—D*	*D—E*	*E—F*	*F—G*
Shift (Stellar Magn.)	+0.16	+0.15	+0.25	+0.42	+.0.19	+0.18.

Interpreting the shifts as variations in the intensity of illumination (which is a pure formal interpretation), we can express the brightness, or rather the photographic effect of each star on each plate in a uniform system of magnitudes — say, in the system of group *A*; the result is given in table 9 for the stars *a* and *b*; the remaining stars cannot be used, for their provisional magnitudes were found on the assumption of a certain form of the curve of interpolation for a part of the plates (represented by groups *E*, *F* and *G*).

Table 9.

Group	Star *a* Photogr. Effect St. magn. (*m*)	Star *a* $\triangle_0$	Star *b* Photogr. Effect St. magn. (*m*)	Star *b* $\triangle_0$
A	0.00	0.65	+1.11	0.14
B	—0.16	0.78	+0.95	0.16
C	—0.31	0.94	+0.80	0.19
D	—0.56	1.13	+0.55	0.38
E	—0.98	1.56	+0.13	0.49
F	—1.17	1.75	—0.06	0.61
G	—1.35	1.98	—0.24	0.65

The form of the relation between m and $\triangle_0$ representing satisfactorily the values of table 9 is to be found. A parabolic function $m = a + b\triangle_0 + c\triangle_0^2$ and the formula $m = a + b \log(i + i_0)$[1]) (i denotes the brightness, $m = -2{,}5 \log i$), proved to be unsatisfactory. The best result gave a hyperbola

$$m = a + b\triangle_0 + \frac{c}{\triangle_0} \quad (5)$$

with the constants $a = +0.422$; $b = -0{,}923$; $c = +0{,}094$. This formula was assumed in the reductions with a slight alteration to account for the different magnitude-density gradient of the different plates. Thus the final form of the interpolation formula used was:

$$m_1 - m_2 = K\left[0{,}923(\triangle_2 - \triangle_1) + 0{,}094\left(\frac{1}{\triangle_1} - \frac{1}{\triangle_2}\right)\right] =$$
$$= K[F(\triangle_1) - F(\triangle_2)] \quad . \; . \; . \quad (6).$$

$\triangle_1$ and $\triangle_2$ are the values of $\triangle_0$ for two stars, whose magnitude difference $m_1 - m_2$ is to be determined. To facilitate the computation, a table of the function $F(\triangle_0) = -0{,}923\,\triangle_0 + \frac{0{,}094}{\triangle_0}$ was constructed, an abbreviated sample of which is given in table 10.

Table 10.

$\triangle_0$	$F(\triangle_0)$	$\triangle_0$	$F(\triangle_0)$
0.10	+0.848	1.0	—0.829
0.12	.672	1.1	0.930
0.14	.549	1.2	1.030
0.16	.442	1.3	1.128
0.18	.359	1.4	1.225
0.20	.285	1.5	1.322
0.25	.145	1.6	1.418
0.30	+0.036	1.7	1.514
0.40	—0.134	1.8	1.609
0.50	.274	1.9	1.705
0.60	.397	2.0	1.799
0.70	.512	2.1	1.893
0.80	.620	2.2	—1.988
0.90	—0.727		

As to the factor K, this was determined separately for each plate from the average values of $\triangle_0$ for the stars a and b, assuming the magnitude difference of these stars equal to $m_b - m_a = 1.11 + \overline{\triangle m_b} - \overline{\triangle m_a}$, $\overline{\triangle m_b}$ and $\overline{\triangle m_a}$ denoting the average deviations due to inequality of diameter and absorption; these quantities, as well as the values of $\triangle_0$ were taken from table 7. Table 11 contains the result.

1) This formula is based on the assumption of an additive effect of the brightness of the background i_0 and the brightness of the star i.

Table 11.

Negative, Exposures used	$m_b - m_a$	$\overline{\triangle_b}$	$\overline{\triangle_a}$	K	Average $K \pm$ p. e.	D_0 (Zero Point)
3 II$_{1-6}$	1.162	0.180	0.937	1.034	1.034±0.040	0.936
5 I$_{1-6}$	1.143	0.150	0.780	1.053	1.050±0.031	0.781
5 II$_{1-6}$	1.134	0.192	0.938	1.049		0.916
6 I$_{1-4}$.6 II$_2$	1.152	0.548	1.716	0.962	0.957±0.022	1.683
6 I$_{5}$.6 II$_{1.3-6}$	1.182	0.722	1.977	0.952		1.914
7 I$_{1-6}$	1.077	0.508	1.583	0.961	0.982± 0.022	1.583
7 II$_{1-3}$	1.134	0.640	1.763	1.003		1.780
8 I	1.097	0.570	1.957	0.785	0.798±0.018	1.955
8 II	1.095	0.693	2.057	0.812		2.081
10	1.153	0.340	1.000	1.447	1.447±0.080	0.960
11 I, II	1.158	0.415	1.180	1.260		
12 I, II	1.134	0.375	1.080	1.385	1.295±0.038	1.126
13 I, II	1.132	0.375	1.175	1.239		
16	1.110	0.190	0.780	1.207	1.207±0.069	0.769

The last column gives the zero point for the negatives (or groups of negatives), i. e. the value of the density $\triangle_0 = D_0$ for which the magnitude would be equal to the normal[1]) magnitude of the star a, assumed as zero. The probable error of the value of K was computed à priori from the probable error of the measured difference of magnitudes $m_b - m_a$; the method of determining the probable error will be discussed later on.

8. Magnitudes of the comparison stars.

For the same grouping of the exposures the definitive magnitudes of the comparison stars were determined. If the average value of $\triangle_0$ for a certain star is $\overline{\triangle_s}$, and if $\overline{\triangle m_s}$ represents the average deviation from the normal value, then the normal magnitude of the star will be given by equation (6) as

$$m_s = K[F(\overline{\triangle_s}) - F(D_0)] - \overline{\triangle m_s} \quad . \; . \; . \; (7),$$

K and D_0 being taken from the 6th and 7th columns of table 11 respectively. In this way table 12 was obtained. The weight was chosen somewhat arbitrarily, taking into account chiefly the number of independent photographs groups) and in a

1) Corrected for the effect of diameter and absorption.

Table 12.
Magnitudes of Comparison Stars.

Negative, Exposures used	Star b'				Star c			
	$\overline{\Delta_0}$	$-\overline{\Delta m}$	m	Weight	$\overline{\Delta_0}$	$-\overline{\Delta m}$	m	Weight
3 II$_{1-6}$	0.353	—0.048	0.686	1	0.168	+0.002	1.211	1
5 I$_{1-6}$	0.293	—0.013	0.669	1	0.110	—0.028	1.399	1
5 II$_{1-6}$	0.352	—0.003	0.723	1	0.158	+0.003	1.257	1
6 I$_{1-4}$. II$_2$	0.900	+0.013	0.751	2	0.452	+0.001	1.236	4
6 II$_{1,3-6}$. I$_5$	1.185	—0.013	0.665	2	0.643	+0.016	1.237	4
7 I$_{1-6}$	0.900	+0.039	0.703	2	0.450	+0.002	1.178	4
7 II$_{1-3}$	1.065	+0.004	0.686	2	0.603	0.000	1.168	4
8 I$_{1-3}$	1.077	+0.016	0.694	2	0.617	—0.003	1.066	4
8 II$_{1-3}$	1.163	—0.022	0.682	2	0.627	—0.014	1.140	4
10	0.660	+0.045	0.511	1	0.320	+0.052	1.195	1
11. 12. 13	0.567	+0.022	0.803	2	0.365	+0.024	1.165	2
16	0.330	+0.072	0.762	1	0.130	+0.012	1.457	1
Weighted Mean (All)			0.696				1.192	
P. Error			± 0.012				± 0.011	
Mean of Plates 6, 7, 8.			0.697				1.171	
P. E.			± 0.008				± 0.017	

Negative, Exposures used	Star g				Star k			
	$\overline{\Delta_0}$	$-\overline{\Delta m}$	m	Weight	$\overline{\Delta_0}$	$-\overline{\Delta m}$	m	Weight
3 II$_{1-6}$	0.618	+0.012	0.370	2	0.340	+0.037	0.794	1
5 I$_{1-6}$	0.502	+0.030	0.368	2	0.300	—0.015	0.652	1
5 II$_{1-6}$	0.592	+0.044	0.415	2	0.362	+0.014	0.722	1
6 I$_{1-4}$. II$_2$	1.256	+0.022	0.417	2	0.865	+0.003	0.776	2
6 II$_{1,3-6}$. I$_5$	1.567	+0.061	0.384	2	1.203	+0.044	0.704	2
7 I$_{1-6}$	1.187	+0.023	0.401	2	0.823	—0.085	0.660	2
7 II$_{1-3}$	1.540	+0.083	0.307	2	1.020	—0.056	0.672	2
8 I$_{1-3}$	1.443	—0.028	0.362	2	1.063	+0.047	0.736	2
8 II$_{1-3}$	1.517	+0.024	0.453	2	1.150	+0.042	0.756	2
10	0.770	—0.030	0.259	1	0.560	+0.002	0.639	1
11. 12. 13	0.858	+0.022	0.377	2	0.605	—0.044	0.672	2
16	0.540	+0.125	0.444	1	0.300	—0.064	0.688	1
Weighted Mean (All)			0.382				0.704	
P. Error			± 0.009				± 0.009	
Mean of Plates 6, 7, 8.			0.387				0.717	
P. E.			± 0.014				± 0.011	

smaller degree — the number of exposures and accuracy of measurement within each group; the latter depends in a high degree upon the value of $\triangle_0$, as may be inferred from formula (6) or table 10; thus the measures for *c*, the faintest comparison star, are of a very low degree of accuracy for the negatives 3, 5 and 16; for the brighter stars, e. g. for *g*, the accuracy is greater and more uniform.

The weighted mean magnitudes, computed from all plates and given at the foot of table 12, were adopted finally; they differ little from the provisional magnitudes of the comparison stars, adopted in section 5. For comparison the average for the six best negatives (6 I.II; 7 I.II; 8 I.II) is given in the same table; no systematic difference between both systems of magnitudes can be perceived; this indicates that the formula of interpolation adopted well fits its purpose for a rather wide range of the photographic density difference $\triangle_0$.

9. **Probable Errors.**

It seemed advisable to estimate the size of the probable error to be expected from various sources; since the method of reduction does not depend upon the accuracy of this estimate, we shall only briefly state the result. Table 13 gives a summary of the various probable errors.

Table 13. Probable Errors.

1) Microphotometer Readings. In Units of the Wedge.		
P. E. of one measure of density of a stellar image		± 0.031 [1]
„ „ „ „ „ of the background		± 0.033
„ of the value of $\triangle_0$ for a single exposure of Neptune (4 measures of one image), if the number of exposures of a group is equal to	1 . . .	± 0.020
	2 . . .	± 0.016
	3 . . .	± 0.015
	6 . . .	± 0.013
„ of $\triangle_0$ for a single exposure of the comp. stars (2 measures), if the number of exposures is	1 . . .	± 0.027
	2 . . .	± 0.022
	3 . . .	± 0.020
	6 . . .	± 0.018

1) Somewhat overestimated, since the systematic difference between the two readings was not taken into account.

Table 13. Continued.

2) Probable error of the sensitiveness of the plate. Stellar magnitudes[1]	± 0.024
3) Probable error of the deviation $\triangle m$, depending upon 1 distance (Average for all stars; for individual values see table 6) ($d = 0.5$ mm)	± 0.020
2 distances	± 0.014
4 "	± 0.010
12 "	± 0.006
4) Probable error of the magnitude of Neptune, arising from the error in the constant K (scale-error); stellar magnitudes[2]:	
Plate 3; 10	± 0.013
5	± 0.009
6; 7; 8	± 0.004
11; 12; 13	± 0.006
16	± 0.018

All these errors are expressed in stellar magnitudes except those of the photometric measures; to convert the units of the wedge into stellar magnitudes, we shall differentiate formula (6); we obtain

$$\delta m = K\delta \triangle_0 \left[0,923 + \frac{0,094}{\triangle_0^2}\right] \quad . \ . \ . \ (8);$$

table 14 contains the values $\frac{\delta m}{K\delta\triangle_0}$, with the aid of which and of the given values of K it is easy to find the equivalent of the photometric errors expressed in stellar magnitudes.

Table 14.

$\triangle_0 =$	0.10	0.15	0.20	0.30	0.40	0.60	0.80	1.00	2.00
$\frac{\delta m}{K\delta\triangle_0} =$	10,3	5,0	3,2	2,0	1,5	1,2	1,1	1,0	0,95.

The rapid increase of the uncertainty of the measures with decreasing $\triangle_0$ is clearly shown by the table.

1) A systematic influence upon the whole group of neighbouring images is included.

2) Were the average magnitude of the comparison stars equal to the magnitude of Neptune, this error would be zero. For different plates the actually adopted values differ, because the fainter comparison stars received on the different plates not equal weight.

With the aid of the tables 13 and 14 the probable error of a magnitude determination can be computed à priori; as will be shown later on, for Neptune the result is not far from the directly determined probable error.

10. Derivation of the photographic brightness of Neptune and discussion of the results.

Let $\triangle_n$ and $\triangle_s$ denote the values of the differential density $\triangle_0$ for Neptune and for a comparison star ($s = a$; b; b'; c; g; k), m_s the normal magnitude[1]), $\triangle m_s$ the deviation of the magnitude of the star and p_s the weight of one magnitude determination of the star; $\triangle_n$, $\triangle_s$ and $\triangle m_s$ must refer to the same exposure; then the magnitude of Neptune will be determined by the following expression:

$$m_n = K[F(\triangle_n) - \overline{F(\triangle_s)}] + \overline{m_s} + \overline{\triangle m_s} \quad . . . (9),$$

where

$$\left.\begin{aligned} \overline{m_s} &= \frac{\Sigma m_s p_s}{\Sigma p_s}, \\ \overline{\triangle m_s} &= \frac{\Sigma \triangle m_s p_s}{\Sigma p_s} \text{ and} \\ \overline{F(\triangle_s)} &= \frac{\Sigma F(\triangle_s) \cdot p_s}{\Sigma p_s} \end{aligned}\right\} s = a,\ b,\ b',\ .\ .\ .\ k.$$

The weight p_s must depend upon the probable error of the

Table 15.
A Priori Probable Errors of the Measure of One Image.

Star	n	c	b	b′	a	g	k
	Stellar Magnitudes						
Negative							
3 II	±0.048	±0.10	±0.073	±0.041	±0.031	±0.032	±0.043
5 I	0.041	0.094	0.086	0.043	0.031	0.035	0.046
5 II	0.041	0.090	0.061	0.040	0.030	0.032	0.038
6 I.II; 7 I.II; 8 I.II	0.029	0.035	0.036	0.035	0.031	0.032	0.032
10; 11; 12; 13	0.040	0.055	0.057	0.043	0.037	0.043	0.050
16	0.050	0.14	0.07	0.050	0.030	0.035	0.050

1) Given in table 12 as the weighted mean, and adopted for b and a: $m_b = 1.110$ and $m_a = 0.000$ respectively.

magnitude of the star. In table 15 the p. e. found a priori from tables 13 and 14 [1]) are given.

On the basis of this table the following weights for the comparison stars were adopted (for all stars with a smaller probable error than for Neptune equal weights were assumed):

Table 16.
Weights of Comparison Stars.

Negative	c	b	b′	a	g	k
3	0.2	0.3	1	1	1	1
5 I	0.2	0.3	1	1	1	1
5 II	0.3	0.5	1	1	1	1
6, 7, 8	1	1	1	1	1	1
10, 11, 12, 13	0.5	0.5	1	1	1	0.5
16	0.2	0.5	1	1	1	1

Table 17.
A priori computed Probable Error of the Magnitude of Neptune, determined from one exposure according to formula (9).

Negative	P. E. St. Mg.	Weight
3 II	±0.055	2
5 I.II	±0.048	2
6 I.II	±0.030	5
7 I.II	±0.031	5
8 I.II	±0.025	8
10	±0.067	1
11—13	±0.053	2
16	±0.069	1

These quantities together with tables 13 and 14 enabled to compute the total probable error, including the scale-error; for the latter error a preliminary value of Neptune's magnitude was needed; it was assumed equal to $+0{,}84$ ($a = 0.00$). The result is given in the following table.

The final result for the magnitude of Neptune is contained in table 18; the table is subdivided into two parts, the first containing the results of the best plates — № 6, 7 and 8, and the second — the result of the remaining plates. The column m_n gives the magnitude of Neptune reduced to a distance from the earth corresponding to March 22, when log ϱ was equal to 1.4683; the reduction for varying distance was sensible only for the observations of April 8—23. The probable error of the first set of negatives is considerably smaller than of the second set; assuming for the first set a weight 7, for the second the weight 1, we obtain the following

1) Since relative probable errors were desired, the scale-error was not taken into account, and for the data of table 14 *K* was assumed equal to unity.

average value of the magnitude of Neptune as determined from all plates:

$$m_n = +0.824 \pm 0.006$$

(System of magnitudes indicated at the head of table 18).

Table 18.
Differential Magnitudes of Neptune.

$a = B.D.\ 16^\circ 1901$ assumed equal to $0^{mg}.000$ } According to the Göttingen Actinometrie.
$b = B.D.\ 17^\circ 2007$ „ „ „ $+1.110$ }

First Series.

Negative, Date, Sidereal Time	$\overline{\triangle m_s}$ St. Mg.	Comparison Stars [1])	Correction for Superposed Images. St. Mg.	m_n St. Mg.	Deviation. St. Mg. × 1000
1922				+	
6 I 1 March 22, 8h.5	—0.006	c, b, b′, a, g, k	+0.002	0.848	+24
2 9.0	„	c, b, b′, a, g	+0.002	.936	+112
3 9.2	„	c, b, b′, a, g, k	+0.002	.902	+78
4 9.5	„	„	+0.013	.852	+28
5 9.8	„	„	—0.009	.758	— 66
6 10.2	„	c, b′, a, g, k	—0.009	.846	+22
6 II 1 10.6	—0.027	c, b, b′, a, g, k	+0.002	.752	—72
2 10.8	„	„	+0.008	.758	—66
3 11.1	„	„	+0.002	.822	— 2
4 11.5	„	„	+0.002	.811	—13
5 11.7	„	„	—0.009	.841	+17
6 12.0	„	„	—0.009	.833	+ 9
7 I 1 March 23, 8.6	+0.002	b, b′, a, g, k	0.	.852	+28
2 8.8	„	c, b, b′, a, g, k	„	.825	+ 1
3 11.6	„	„	„	.758	—66
4 11.8	„	b, b′, a, g, k	„	.773	—51
5 11.0	„	c, b, b′, a, g, k	„	.867	+43
7 II 1 12.1	+0.002	c, b, a, g, k	„	.780	—44
2 12.4	„	c, b, b′, a, g, k	„	.827	+ 3
3 12.7	„	„	„	.831	+ 7
8 I 2 March 24, 8.8	—0.017	„	„	.813	—11
4 9.1	„	„	„	.864	+40
6 9.3	„	„	„	.837	+13
8 II 1 9.6	—0.011	„	„	.809	—15
4 9.9	„	„	„	.754	—70
5 10.1	„	„	„	+0.799	—25
			Average	+0.821	± 0.006 (p.e.)

1) Of equal weight.

Table 18. Continued.
Second Series.

Negative, Date, Sidereal Time	$\overline{\triangle m_s}$ St. Mg.	Comparison Stars and their Weights, indicated by Coefficients	m_n[1])	Deviation. St. Mg. × 1000
1922			+	
3 II 1 March 20, 10h.7	+0.003	0,2c; 0,3b; b′, a, g, k	0.887	+ 63
2 10.4	+0.003	„	0.904	+ 80
3 10.0	+0.003	0,3b; b′, a, g, k	0.809	— 15
4 9.7	+0.003	0,2c; 0,3b; b′, a, g, k	1.154	+330
5 9.3	+0.015	0,2c; 0,3b; b′, a, g	0.927	+103
6 9.1	+0.003	0,3b; b′, a, g, k	1.021	+197
7 11.0	+0.015	0,2c; b′, a, g	0.928	+104
5 I 1 March 21, 8.4	+0.004	0,2c; 0,3b; b′, a, g, k	0.724	—100
2 8.7	0.000	0,3b; b′, a, g	0.750	— 74
3 9.0	„	0,2c; 0,3b; b′, a, g	0.716	—108
4 9.3	+0.004	0,2c; 0,3b; b′, a, g, k	0.759	— 65
5 9.7	„	„	1.038	+212
6 10.3	„	„	1.027	+203
5 II 1 10.7	—0.015	0,3c; 0,5b; b′, a, g, k	0.776	— 48
2 11.0	„	„	0.818	— 6
3 11.2	„	„	0.918	+ 94
4 11.5	„	0,5b; b′, a, g, k	0.909	+ 85
5 11.8	„	0,3c; 0,5b; b′, a, g, k	0.752	— 72
6 12.1	„	„	0.670	—154
10. April 8, 11.5	—0.024	0,5c; 0,5b; b′, a, g; 0,5k	0.882	+ 58
11 I April 12, 10.8	—0.008	„	0.807	— 17
11 II 11.0	—0.002	„	0.834	+ 10
12 I 11.6	+0.021	„	0.767	— 57
12 II 11.3	—0.059	„	0.662	—162
13 I 11.9	+0.017	„	0.731	— 93
13 II 12.3	—0.037	„	0.691	—133
16, 1 April 23, 12.5	—0.033	0,2c; 0,5b; b′, a, g, k	0.945	+121
4 12.8	„	„	0.709	—115
		Average	0.840	± 0.016 (p.e.)

The deviations of the single observations from this average value are given in the last column of table 18 and are expressed in thousandths of a stellar magnitude.

1) Reduced to log $\varrho = 1.4683$, ϱ being the distance from the earth.

The „correction for superposed images“ deserves some explanation. The crowding of several exposures upon the same plate increases the probability of the fainter stars being superposed on the extrafocal images of Neptune[1]). The number of the different points of the background of the sky superposed upon a certain image k is equal to the number of exposures, and the coordinates of these points are found by subtracting from the coordinates of Neptune the differences $\alpha_i - \alpha_k$ and $\delta_i - \delta_k$ (or the differential right ascension and declination of all exposures, the exposure k being taken as origin). The image i whose effect upon k must be determined we shall call the „active image“. It may be remarked, that the effect of i upon k is not equal to the effect of k upon i.

All stars in the neighbourhood of Neptune, found on the Carte du Ciel, Bordeaux, 16° № 69, were examined for superposition in the case of the negatives 6, 7 and 8; the diameter of the central image upon these negatives was on the average 0,33 mm and the diameter of the circle measured on the microphotometer 0,11 mm; in the scale of the Carte du Ciel these values must be increased 8.7-fold. It was found that a superposition took place only for the negative 6, when a star of the estimated magnitude 10.5, coordinates $x = -25.95$, $y = +59.45$, covered upon several images $1/4^{th}$ of the field of the microphotometer. From the stars a and k the systematic difference of the magnitudes of the Carte du Ciel and our adopted system was found equal to 6.5 and 6.8 respectively, and a mean of 6.6 was adopted; this gives the difference of magnitudes between the said star and Neptune equal to 3.1, which corresponds to 0.058 of the brightness of Neptune; $1/4^{th}$ of this quantity was taken and was multiplied by a provisional value of Schwarzschild's exponent $p = 0.75$[2]); finally the effect of superposition was adopted equal to $0.011\,\tau$, τ being the ratio of the exposures of the active and the given image. The following table was obtained:

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1) For the comparison stars the same danger exists, but it has not been taken into account because its effect is generally small and the effect will be lost in forming the average magnitude of all six stars.

2) Determined for the same plate.

Image	Deviation of Brightness due to Superposition	Correction for Superposition. St. Mg.
6 I 1, 2, 3 6 II 1, 3, 4	+0.011	+0.002
6 I 4	+0.022	+0.013
6 II 2	+0.017	+0.008
6 I.II 5, 6	0.000	—0.009
Average	+0.009	0.000

Since the background was affected by the same star, the correction was applied so that its average value would be equal to zero.

The sign and numerical value of the correction expressed in stellar magnitudes is equal to the deviation in units of the brightness of Neptune. The entire process of finding the correction for superposed images is of a somewhat inconvenient character, and precise results for this correction cannot be expected; but owing to the smallness of the correction in the actual case the indeterminateness of the problem has no consequences. The chief result of the examination of the effect of overlapping images is that this effect was practically negligeable for the negatives 6, 7 and 8. Owing to the low accuracy of the measures of the remaining negatives, they were considered not to be worth of the labour of applying such a minute correction.

The probable errors computed from the deviations in the last column of table 18, were:

for the plates 6; 7; 8 . . . . p. e. = ± 0.032 st. mg.
„ „ „ 3; 5; 10—13; 16 . . . p. e. = ± 0.081 st. mg.

These probable errors may be called the „observed p. e.“; the probable errors to be expected in both cases were, according to table 17, equal to ± 0.029 and ± 0.053 st. mg. respectively. For the first set of measures the agreement between the observed and the a priori computed probable errors is good; but for the second set the observed probable error is considerably greater than the expected one; a better agreement would be obtained if it were assumed that the error due to the variation of the sensitiveness of the plate increased with decreasing density of the images.

The question of the variability of Neptune can be discussed only on the basis of the first set of measures comprising negatives 6, 7 and 8. The close agreement of the observed and expected p. e. seems to be a test against any variability; however, the a priori expected p. e. may be somewhat exaggerated, as indicates the first footnote of table 13; and a small amplitude of variation can exist without affecting in a sensible manner the probable deviation from the mean.

The deviations from the mean magnitude of the individual observations of March 22, 23 and 24 are plotted on fig. 2. The first glance at this figure reveals a systematical variation in the magnitude of Neptune during these three consecutive nights; the broken line indicates the hypothetical form of the curve of

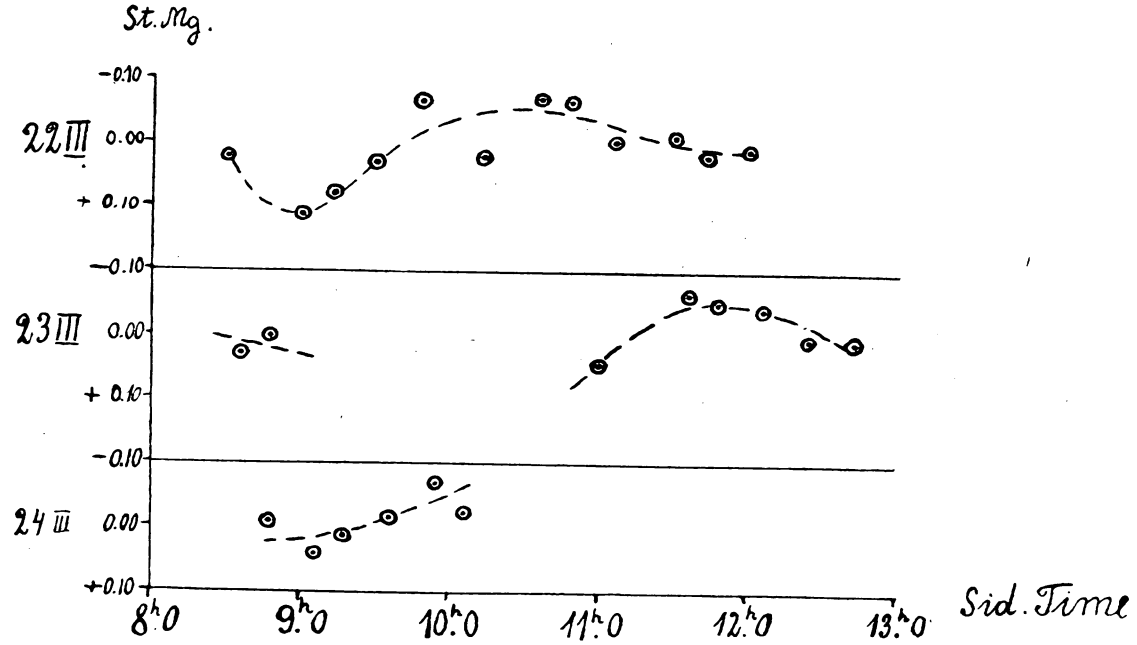

Fig. 2.
Photographic Magnitude of Neptune.

variation; all three curves can be made to agree with a certain adopted short period; but the material is too scant to obtain any reliable result as to the form and period of the variation. The probable deviation of one observation from the hypothetical curves is only ± 0.016 st. mg.; this may be explained by the small number of points, which caused the somewhat arbitrarily drawn curves pass nearer to the individual points: curves of the same amplitude and representing the observations equally well may be drawn, so that the probable deviation would be increased up to ± 0.020 or even more. In any case a variation of the photographic brightness of Neptune of an amplitude of 0.08—0.15 st. mg. and of a short period is indicated by the present observations with a considerable degree of probability.

As to the arrangement of future observations, from the probable errors due to the different sources (see table 13) the conclusion may be drawn that a number of exposures on the same negative greater than 2 increases the accuracy of the observations but little; if the danger of overlapping stellar images and the inconvenience of eliminating it is taken into account, it must be inferred that a number of 2—3 exposures upon the same negative of a star of the brightness of Neptune gives the optimal result; a greater or a smaller number will affect the accuracy of the observations.

October 1922.